Lista 12

MAT01168 – Matemática Aplicada II – 2015/1

Exercício 1. Determine a Série de Laurent das funções abaixo em torno do ponto z_0.

$(a)\ f(z) = \dfrac{e^{-z}}{z^3},\ z_0 = 0$

$(b)\ f(z) = z^{-3}e^{1/z},\ z_0 = 0$

$(c)\ f(z) = \dfrac{1}{z^2+1},\ z_0 = i$

$(d)\ f(z) = \dfrac{\cos z}{(z-\pi)^4},\ z_0 = \pi$

Exercício 2. Todos os círculos abaixos são percorridos no sentido anti–horário. Calcule:

$(a)\ \oint_C e^{1/z}\ dz$, onde $C:\ |z| = 1$

$(b)\ \oint_C \tan(\pi z)\ dz$, onde $C:\ |z| = 1$

$(c)\ \oint_C \tan(\pi z)\ dz$, onde $C:\ |z| = 2$

$(d)\ \oint_C \dfrac{e^z}{\cos z}\ dz$, onde $C:\ |z| = 9/2$

$(e)\ \oint_C \dfrac{e^z}{\cos(\pi z)}\ dz$, onde $C:\ |z-i| = 3/2$

$(f)\ \oint_C \dfrac{30z^2-23z+5}{(2z-1)^2(3z-1)}\ dz$, onde $C:\ |z| = 1$

Exercício 3. Utilize integrais complexas para calcular as integrais abaixo. Mostre os detalhes (passo a passo do método).

$(a)\ \displaystyle\int_0^{2\pi} \frac{1}{7+6\cos\theta}\ d\theta$

$(b)\ \displaystyle\int_0^{2\pi} \frac{1}{5-4\,\mathrm{sen}\,\theta}\ d\theta$

$(c)\ \displaystyle\int_0^{2\pi} \frac{\mathrm{sen}^2\,\theta}{5-4\cos\theta}\ d\theta$

$(d)\ \displaystyle\int_0^{2\pi} \frac{\cos\theta}{13-12\cos(2\theta)}\ d\theta$

$(e)\ \displaystyle\int_{-\infty}^{+\infty} \frac{1}{x^2+1}\ dx$

$(f)\ \displaystyle\int_{-\infty}^{+\infty} \frac{1}{x^4+16}\ dx$

$(g)\ \displaystyle\int_{-\infty}^{+\infty} \frac{x^3}{1+x^8}\ dx$

RESPOSTAS – Em breve

1a. $\displaystyle\sum_{n=0}^{+\infty} \frac{(-1)^n z^{n-3}}{n!} = \frac{1}{z^3} - \frac{1}{z^2} + \frac{1}{2z} - \sum_{n=0}^{+\infty} \frac{(-1)^n z^n}{(n+3)!}$

1b. $\displaystyle\sum_{n=0}^{+\infty} \frac{1}{n! z^{n+3}} = \frac{1}{z^3} + \frac{1}{z^2} + \frac{1}{2z} + \frac{1}{6} + \frac{z}{24} + \frac{z^2}{120} + \cdots$

1c. $\displaystyle\sum_{n=0}^{+\infty}\frac{(-1)^n}{(2i)^{n-1}}(z-i)^{n-1}=\frac{2i}{z-i}-\sum_{n=0}^{+\infty}\frac{i^n}{2^n}(z-i)^n$

1d. $\displaystyle\sum_{n=0}^{+\infty}\frac{(-1)^{n+1}}{(2n)!}(z-\pi)^{2n-4}=-\frac{1}{(z-\pi)^4}+\frac{1}{2(z-\pi)^2}-\frac{1}{24}+\frac{(z-\pi)^2}{6!}-\cdots$

2a. $2\pi i$

2b. $-4i$

2c. $-8i$

2d. $-4\pi i\,\mathrm{senh}(\pi/2)$

2e. $-4i\,\mathrm{senh}(1/2)$

2f. $5\pi i$

3a. $2\pi/\sqrt{13}$

3b. $2\pi/3$

3c. $\pi/4$

3d. 0

3e. π

3f. $\pi/8\sqrt{2}$

3g. 0