Addendum to "Estimation and Forecasting in Seasonal Long Memory Processes"

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1. Introduction and Goals

This report presents an addendum to the paper by Bisognin and Lopes (2011), "Estimation and Forecasting in Seasonal Long Memory Processes", submitted to the Journal of Time Series Analysis. In this work the authors consider the fractional ARIMA process with seasonality s, denoted by SARFIMA $(p, d, q) \times (P, D, Q)_s$, which describes time series with long memory periodical behavior at finite number of spectrum frequencies. The authors present several estimation methods for the parameters of these processes. They also consider the minimum mean squared forecasting error for the complete SARFIMA process. The whole methodology is applied to the water consumption time series, observed in Granada, Spain.

The work presents an extensive Monte Carlo simulation study by analyzing the estimators' performance through their bias, mean squared error and variance values.

The process $\{X_t\}_{t\in\mathbb{Z}}$, is simulated as suggested by Hosking (1981), where $\{\varepsilon_t\}_{t\in\mathbb{Z}}$ is a Gaussian white noise process with zero mean and variance $\sigma_\varepsilon^2 = 1.0$.

It is not our intention to provide a full account of the results in the work here, but solely to provide more material to facilitate its understanding. We refer the reader to the original work for all the information on the mathematical set up, general and specific definitions, simulation methods, conclusions and any other information refereing to the data and tables presented here.

In this addendum we report the estimation results for $d = 0.2 = D$, $\Phi_1, \phi_1 \in \{-0.5, 0.5\}$, $\Theta_1, \theta_1 \in \{-0.5, 0.5\}$, $n \in \{1000, 2000\}$ and $s \in \{4, 6, 12\}$. For the three versions of the semiparametric estimators GPH, BA, R, SR and GPHT, the trimming constant α is such that $\alpha \in \{0.75, 0.8, 0.89\}$.

Both loss functions ρ_j, $j = 1, 2$, for the MM-estimators are chosen as the Tukey Biweighted function (see Yohai, 1987). They are tuned such that the resulting estimates possess 0.50 breakdown point and an efficiency of 85% at the Gaussian model.

We calculate the empirical values for the mean, the bias, the mean square error (mse) and the variance (var). These quantities are given in Tables 1.1 to 1.24. All estimation procedures are based upon 1000 replications.

Tables 1.1 to 1.8 present the semiparametric estimation results for the SARFIMA$(p, d, q) \times (P, D, Q)_s$ processes, when $p = P = 0 = q = Q$, $s \in \{4, 6, 12\}$ and $\alpha \in \{0.75, 0.8, 0.89\}$. Tables 1.9 to 1.24 present the parametric estimation results for the SARFIMA$(p, d, q) \times (P, D, Q)_s$ processes, when $s \in \{4, 6, 12\}$, $\min\{p, P, q, Q\} \neq 0$, $\phi_1, \Phi_1 \in \{-0.5, 0.5\}$ and $\theta_1, \Theta_1 \in \{-0.5, 0.5\}$.

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References

C. Bisognin and S.R.C. Lopes (2011). "Estimation and Forecasting in Seasonal Long Memory Processes". Submitted to *Journal of Times Series Analysis*.

Table 1.1: Semiparametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ processes, when $p=P=0=q=Q$, $d=0.2=D$, $s=4$, $n\in\{1000,2000\}$, $\ell=2$ and $\alpha=0.75$.

	$n=1000$						$n=2000$					
	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$
	GPHLS		GPHMM		GPHLTS		GPHLS		GPHMM		GPHLTS	
mean	0.1748	0.2854	0.2195	0.2136	0.1989	0.2520	0.1480	0.2890	0.1886	0.2329	0.1679	0.2643
bias	-0.0252	0.0854	0.0195	0.0136	-0.0011	0.0520	-0.0520	0.0890	-0.0114	0.0329	-0.0321	0.0643
mse	0.0454	0.0784	0.0815	0.1374	0.0493	0.0834	0.0730	0.1061	0.1293	0.1799	0.0835	0.1198
var	0.0448	0.0711	0.0812	0.1373	0.0493	0.0808	0.0703	0.0983	0.1293	0.1790	0.0826	0.1157
	RLM		RMM		RLTS		RLM		RMM		RLTS	
mean	0.1919	0.2516	0.2361	0.1835	0.2024	0.2391	0.1692	0.2581	0.1965	0.2201	0.1782	0.2484
bias	-0.0081	0.0516	0.0361	-0.0165	0.0024	0.0391	-0.0308	0.0581	-0.0035	0.0201	-0.0218	0.0484
mse	0.0451	0.0738	0.0847	0.1377	0.0499	0.0821	0.0694	0.0971	0.1314	0.1811	0.0851	0.1191
var	0.0451	0.0712	0.0835	0.1375	0.0499	0.0807	0.0685	0.0938	0.1315	0.1809	0.0848	0.1169
	SRLM		SRMM		SRLTS		SRLM		SRMM		SRLTS	
mean	0.2110	0.1975	0.2158	0.1888	0.2127	0.1947	0.2034	0.2017	0.2130	0.1909	0.2060	0.1989
bias	0.0110	-0.0025	0.0158	-0.0112	0.0127	-0.0053	0.0034	0.0017	0.0130	-0.0091	0.0060	-0.0011
mse	0.0268	0.0420	0.0408	0.0656	0.0364	0.0579	0.0405	0.0548	0.0659	0.0904	0.0552	0.0747
var	0.0267	0.0420	0.0406	0.0655	0.0363	0.0579	0.0405	0.0548	0.0658	0.0904	0.0552	0.0748
	BALM		BAMM		BALTS		BALM		BAMM		BALTS	
mean	0.2253	0.1692	0.2231	0.1750	0.2189	0.1823	0.2251	0.1701	0.2205	0.1797	0.2153	0.1856
bias	0.0253	-0.0308	0.0231	-0.0250	0.0189	-0.0177	0.0251	-0.0299	0.0205	-0.0203	0.0153	-0.0144
mse	0.0271	0.0421	0.0411	0.0657	0.0361	0.0569	0.0413	0.0561	0.0651	0.0894	0.0564	0.0760
var	0.0265	0.0412	0.0406	0.0652	0.0357	0.0567	0.0408	0.0553	0.0648	0.0891	0.0562	0.0759
	GPHTLM		GPHTMM		GPHTLTS		GPHTLM		GPHTMM		GPHTLTS	
mean	0.1773	0.2819	0.2169	0.2129	0.1887	0.2644	0.1242	0.3181	0.1735	0.2473	0.1249	0.3144
bias	-0.0227	0.0819	0.0169	0.0129	-0.0113	0.0644	-0.0758	0.1181	-0.0265	0.0473	-0.0751	0.1144
mse	0.0755	0.1309	0.1056	0.1817	0.0740	0.1298	0.1117	0.1599	0.1778	0.2555	0.1149	0.1664
var	0.0751	0.1243	0.1054	0.1817	0.0739	0.1258	0.1060	0.1461	0.1772	0.2536	0.1094	0.1535

Table 1.2: Semiparametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ processes, when $p=P=0=q=Q$, $d=0.2=D$, $s=4$, $n\in\{1000,2000\}$, $\ell=2$ and $\alpha=0.89$.

	$n=1000$						$n=2000$					
	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$
	GPHLS		GPHMM		GPHLTS		GPHLS		GPHMM		GPHLTS	
mean	0.2187	0.2168	0.2109	0.2091	0.2162	0.2142	0.2101	0.2082	0.2079	0.2012	0.2101	0.2077
bias	0.0187	0.0168	0.0109	0.0091	0.0162	0.0142	0.0101	0.0082	0.0079	0.0012	0.0101	0.0077
mse	0.0016	0.0020	0.0033	0.0041	0.0020	0.0024	0.0008	0.0009	0.0018	0.0019	0.0010	0.0011
var	0.0013	0.0017	0.0032	0.0040	0.0018	0.0022	0.0007	0.0008	0.0017	0.0019	0.0009	0.0010
	RLS		RMM		RLTS		RLS		RMM		RLTS	
mean	0.2144	0.2132	0.2091	0.2088	0.2130	0.2103	0.2079	0.2065	0.2069	0.2003	0.2073	0.2069
bias	0.0144	0.0132	0.0091	0.0088	0.0130	0.0103	0.0079	0.0065	0.0069	0.0003	0.0073	0.0069
mse	0.0015	0.0019	0.0032	0.0040	0.0019	0.0022	0.0007	0.0009	0.0018	0.0020	0.0009	0.0012
var	0.0013	0.0017	0.0031	0.0040	0.0017	0.0021	0.0007	0.0008	0.0017	0.0020	0.0009	0.0011
	SRLM		SRMM		SRLTS		SRLM		SRMM		SRLTS	
mean	0.2039	0.2049	0.2039	0.2043	0.2038	0.2043	0.2021	0.2034	0.2017	0.2022	0.2017	0.2023
bias	0.0039	0.0049	0.0039	0.0043	0.0038	0.0043	0.0021	0.0034	0.0017	0.0022	0.0017	0.0023
mse	0.0008	0.0011	0.0013	0.0017	0.0011	0.0014	0.0004	0.0005	0.0007	0.0008	0.0006	0.0007
var	0.0008	0.0011	0.0013	0.0017	0.0011	0.0014	0.0004	0.0005	0.0007	0.0008	0.0006	0.0007
	BALM		BAMM		BALTS		BALM		BAMM		BALTS	
mean	0.2008	0.2023	0.2030	0.2030	0.2023	0.2040	0.2002	0.2018	0.2010	0.2017	0.2013	0.2017
bias	0.0008	0.0023	0.0030	0.0030	0.0023	0.0040	0.0002	0.0018	0.0010	0.0017	0.0013	0.0017
mse	0.0008	0.0011	0.0013	0.0017	0.0011	0.0015	0.0004	0.0005	0.0007	0.0008	0.0006	0.0007
var	0.0008	0.0011	0.0013	0.0017	0.0011	0.0014	0.0004	0.0005	0.0007	0.0008	0.0006	0.0007
	GPHTLM		GPHTMM		GPHTLTS		GPHTLM		GPHTMM		GPHTLTS	
mean	0.2174	0.2153	0.2109	0.2049	0.2152	0.2117	0.2095	0.2097	0.2056	0.2022	0.2076	0.2088
bias	0.0174	0.0153	0.0109	0.0049	0.0152	0.0117	0.0095	0.0097	0.0056	0.0022	0.0076	0.0088
mse	0.0024	0.0030	0.0042	0.0058	0.0025	0.0034	0.0012	0.0015	0.0021	0.0026	0.0014	0.0018
var	0.0021	0.0027	0.0041	0.0058	0.0022	0.0032	0.0011	0.0014	0.0020	0.0026	0.0014	0.0017

Table 1.3: Semiparametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ processes, when $p=P=0=q=Q$, $d=0.2=D$, $s=6$, $n\in\{1000,2000\}$, $\ell=2$ and $\alpha=0.75$.

	$n=1000$						$n=2000$					
	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$
	GPHLS		GPHMM		GPHLTS		GPHLS		GPHMM		GPHLTS	
mean	0.2355	0.2209	0.2303	0.2054	0.2352	0.2143	0.2110	0.2234	0.2134	0.2061	0.2148	0.2162
bias	0.0355	0.0209	0.0303	0.0054	0.0352	0.0143	0.0110	0.0234	0.0134	0.0061	0.0148	0.0162
mse	0.0046	0.0036	0.0077	0.0060	0.0050	0.0039	0.0045	0.0098	0.0090	0.0179	0.0054	0.0111
var	0.0033	0.0031	0.0067	0.0060	0.0038	0.0037	0.0044	0.0092	0.0088	0.0179	0.0052	0.0109
	RLM		RMM		RLTS		RLM		RMM		RLTS	
mean	0.2272	0.2154	0.2251	0.2015	0.2289	0.2107	0.2109	0.2147	0.2125	0.2034	0.2131	0.2122
bias	0.0272	0.0154	0.0251	0.0015	0.0289	0.0107	0.0109	0.0147	0.0125	0.0034	0.0131	0.0122
mse	0.0039	0.0034	0.0069	0.0057	0.0044	0.0039	0.0045	0.0091	0.0089	0.0178	0.0051	0.0104
var	0.0032	0.0031	0.0063	0.0057	0.0036	0.0038	0.0044	0.0089	0.0087	0.0178	0.0050	0.0103
	SRLM		SRMM		SRLTS		SRLM		SRMM		SRLTS	
mean	0.2075	0.2008	0.2058	0.1995	0.2080	0.2019	0.2041	0.2012	0.2063	0.1980	0.2051	0.2001
bias	0.0075	0.0008	0.0058	-0.0005	0.0080	0.0019	0.0041	0.0012	0.0063	-0.0020	0.0051	0.0001
mse	0.0020	0.0018	0.0032	0.0028	0.0026	0.0025	0.0027	0.0051	0.0042	0.0084	0.0038	0.0072
var	0.0019	0.0018	0.0031	0.0028	0.0025	0.0025	0.0027	0.0051	0.0042	0.0084	0.0037	0.0072
	BALM		BAMM		BALTS		BALM		BAMM		BALTS	
mean	0.2007	0.1963	0.2027	0.1973	0.2047	0.2000	0.2039	0.1923	0.2058	0.1951	0.2058	0.1951
bias	0.0007	-0.0037	0.0027	-0.0027	0.0047	0.0000	0.0039	-0.0077	0.0058	-0.0049	0.0058	-0.0049
mse	0.0019	0.0018	0.0030	0.0029	0.0026	0.0024	0.0027	0.0052	0.0041	0.0083	0.0037	0.0072
var	0.0019	0.0018	0.0030	0.0029	0.0026	0.0024	0.0027	0.0052	0.0041	0.0082	0.0037	0.0071
	GPHTLM		GPHTMM		GPHTLTS		GPHTLM		GPHTMM		GPHTLTS	
mean	0.2356	0.2205	0.2277	0.2049	0.2353	0.2162	0.2074	0.2316	0.2095	0.2115	0.2062	0.2284
bias	0.0356	0.0205	0.0277	0.0049	0.0353	0.0162	0.0074	0.0316	0.0095	0.0115	0.0062	0.0284
mse	0.0064	0.0057	0.0094	0.0091	0.0066	0.0058	0.0068	0.0147	0.0114	0.0242	0.0071	0.0155
var	0.0051	0.0053	0.0086	0.0091	0.0054	0.0056	0.0067	0.0137	0.0114	0.0241	0.0070	0.0148

Table 1.4: Semiparametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ processes, when $p=P=0=q=Q$, $d=0.2=D$, $s=6$, $n\in\{1000,2000\}$, $\ell=2$ and $\alpha=0.8$.

	$n=1000$						$n=2000$					
	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$
	GPHLS		GPHMM		GPHLTS		GPHLS		GPHMM		GPHLTS	
mean	0.2261	0.2190	0.2202	0.2048	0.2233	0.2140	0.2150	0.2135	0.2108	0.2044	0.2144	0.2120
bias	0.0261	0.0190	0.0202	0.0048	0.0233	0.0140	0.0150	0.0135	0.0108	0.0044	0.0144	0.0120
mse	0.0030	0.0033	0.0052	0.0059	0.0032	0.0036	0.0015	0.0016	0.0028	0.0033	0.0017	0.0020
var	0.0024	0.0029	0.0048	0.0059	0.0027	0.0034	0.0013	0.0014	0.0026	0.0033	0.0015	0.0018
	RLM		RMM		RLTS		RLM		RMM		RLTS	
mean	0.2196	0.2139	0.2158	0.2043	0.2200	0.2091	0.2115	0.2107	0.2090	0.2028	0.2112	0.2106
bias	0.0196	0.0139	0.0158	0.0043	0.0200	0.0091	0.0115	0.0107	0.0090	0.0028	0.0112	0.0106
mse	0.0027	0.0031	0.0049	0.0058	0.0029	0.0034	0.0014	0.0014	0.0027	0.0033	0.0015	0.0018
var	0.0023	0.0029	0.0047	0.0058	0.0025	0.0034	0.0012	0.0013	0.0026	0.0033	0.0014	0.0017
	SRLM		SRMM		SRLTS		SRLM		SRMM		SRLTS	
mean	0.2059	0.2010	0.2059	0.2010	0.2056	0.2016	0.2029	0.2031	0.2023	0.2024	0.2029	0.2034
bias	0.0059	0.0010	0.0059	0.0010	0.0056	0.0016	0.0029	0.0031	0.0023	0.0024	0.0029	0.0034
mse	0.0014	0.0017	0.0020	0.0028	0.0018	0.0024	0.0008	0.0009	0.0013	0.0014	0.0011	0.0012
var	0.0013	0.0017	0.0020	0.0028	0.0018	0.0024	0.0008	0.0008	0.0013	0.0014	0.0011	0.0012
	BALM		BAMM		BALTS		BALM		BAMM		BALTS	
mean	0.2011	0.1972	0.2047	0.1986	0.2039	0.2000	0.1995	0.2003	0.2010	0.2012	0.2016	0.2025
bias	0.0011	-0.0028	0.0047	-0.0014	0.0039	0.0000	-0.0005	0.0003	0.0010	0.0012	0.0016	0.0025
mse	0.0014	0.0017	0.0020	0.0028	0.0018	0.0023	0.0008	0.0008	0.0013	0.0015	0.0011	0.0012
var	0.0014	0.0017	0.0020	0.0028	0.0018	0.0024	0.0008	0.0008	0.0013	0.0015	0.0011	0.0012
	GPHTLM		GPHTMM		GPHTLTS		GPHTLM		GPHTMM		GPHTLTS	
mean	0.2258	0.2184	0.2207	0.2028	0.2244	0.2154	0.2149	0.2156	0.2074	0.2061	0.2128	0.2118
bias	0.0258	0.0184	0.0207	0.0028	0.0244	0.0154	0.0149	0.0156	0.0074	0.0061	0.0128	0.0118
mse	0.0043	0.0051	0.0063	0.0081	0.0044	0.0051	0.0021	0.0024	0.0038	0.0047	0.0022	0.0028
var	0.0036	0.0048	0.0059	0.0081	0.0038	0.0049	0.0019	0.0022	0.0037	0.0047	0.0020	0.0027

Table 1.5: Semiparametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ processes, when $p=P=0=q=Q$, $d=0.2=D$, $s=6$, $n\in\{1000,2000\}$, $\ell=2$ and $\alpha=0.89$.

	$n=1000$						$n=2000$					
	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$
	GPHLS		GPHMM		GPHLTS		GPHLS		GPHMM		GPHLTS	
mean	0.2195	0.2174	0.2118	0.2066	0.2169	0.2135	0.2105	0.2097	0.2063	0.2068	0.2097	0.2078
bias	0.0195	0.0174	0.0118	0.0066	0.0169	0.0135	0.0105	0.0097	0.0063	0.0068	0.0097	0.0078
mse	0.0017	0.0019	0.0032	0.0041	0.0019	0.0022	0.0008	0.0008	0.0016	0.0018	0.0010	0.0011
var	0.0013	0.0016	0.0031	0.0041	0.0016	0.0020	0.0007	0.0007	0.0016	0.0018	0.0009	0.0010
	RLM		RMM		RLTS		RLM		RMM		RLTS	
mean	0.2148	0.2141	0.2100	0.2052	0.2138	0.2107	0.2081	0.2080	0.2058	0.2062	0.2082	0.2069
bias	0.0148	0.0141	0.0100	0.0052	0.0138	0.0107	0.0081	0.0080	0.0058	0.0062	0.0082	0.0069
mse	0.0014	0.0018	0.0031	0.0041	0.0018	0.0022	0.0007	0.0007	0.0015	0.0018	0.0009	0.0010
var	0.0012	0.0016	0.0030	0.0041	0.0016	0.0021	0.0006	0.0007	0.0015	0.0017	0.0008	0.0010
	SRLM		SRMM		SRLTS		SRLM		SRMM		SRLTS	
mean	0.2042	0.2035	0.2039	0.2035	0.2040	0.2039	0.2025	0.2024	0.2016	0.2029	0.2014	0.2031
bias	0.0042	0.0035	0.0039	0.0035	0.0040	0.0039	0.0025	0.0024	0.0016	0.0029	0.0014	0.0031
mse	0.0008	0.0009	0.0013	0.0015	0.0010	0.0013	0.0004	0.0004	0.0007	0.0007	0.0006	0.0007
var	0.0008	0.0009	0.0012	0.0015	0.0010	0.0013	0.0004	0.0004	0.0007	0.0007	0.0006	0.0007
	BALM		BAMM		BALTS		BALM		BAMM		BALTS	
mean	0.2008	0.2011	0.2022	0.2021	0.2026	0.2035	0.2004	0.2009	0.2009	0.2026	0.2014	0.2030
bias	0.0008	0.0011	0.0022	0.0021	0.0026	0.0035	0.0004	0.0009	0.0009	0.0026	0.0014	0.0030
mse	0.0008	0.0009	0.0012	0.0015	0.0010	0.0013	0.0004	0.0004	0.0007	0.0007	0.0005	0.0007
var	0.0008	0.0009	0.0012	0.0015	0.0010	0.0013	0.0004	0.0004	0.0007	0.0007	0.0005	0.0007
	GPHTLM		GPHTMM		GPHTLTS		GPHTLM		GPHTMM		GPHTLTS	
mean	0.2179	0.2168	0.2119	0.2067	0.2143	0.2148	0.2103	0.2106	0.2063	0.2050	0.2074	0.2104
bias	0.0179	0.0168	0.0119	0.0067	0.0143	0.0148	0.0103	0.0106	0.0063	0.0050	0.0074	0.0104
mse	0.0023	0.0028	0.0038	0.0049	0.0025	0.0032	0.0011	0.0012	0.0018	0.0025	0.0013	0.0016
var	0.0020	0.0026	0.0037	0.0049	0.0022	0.0029	0.0010	0.0011	0.0018	0.0025	0.0012	0.0015

Table 1.6: Semiparametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ processes, when $p=P=0=q=Q$, $d=0.2=D$, $s=12$, $n\in\{1000,2000\}$, $\ell=2$ and $\alpha=0.75$.

	$n=1000$						$n=2000$					
	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$
	GPHLS		GPHMM		GPHLTS		GPHLS		GPHMM		GPHLTS	
mean	0.2350	0.2292	0.2279	0.2179	0.2351	0.2235	0.2188	0.2143	0.2171	0.1962	0.2207	0.2107
bias	0.0350	0.0292	0.0279	0.0179	0.0351	0.0235	0.0188	0.0143	0.0171	-0.0038	0.0207	0.0107
mse	0.0044	0.0047	0.0067	0.0077	0.0049	0.0049	0.0023	0.0031	0.0039	0.0057	0.0026	0.0034
var	0.0032	0.0039	0.0059	0.0074	0.0036	0.0043	0.0019	0.0029	0.0036	0.0057	0.0022	0.0033
	RLM		RMM		RLTS		RLM		RMM		RLTS	
mean	0.2263	0.2227	0.2217	0.2121	0.2275	0.2191	0.2145	0.2103	0.2128	0.1962	0.2169	0.2071
bias	0.0263	0.0227	0.0217	0.0121	0.0275	0.0191	0.0145	0.0103	0.0128	-0.0038	0.0169	0.0071
mse	0.0038	0.0043	0.0063	0.0075	0.0043	0.0044	0.0020	0.0030	0.0038	0.0058	0.0024	0.0032
var	0.0031	0.0037	0.0059	0.0074	0.0036	0.0041	0.0018	0.0028	0.0037	0.0058	0.0021	0.0032
	SRLM		SRMM		SRLTS		SRLM		SRMM		SRLTS	
mean	0.2065	0.2023	0.2062	0.2047	0.2064	0.2052	0.2042	0.1984	0.2057	0.1971	0.2050	0.1982
bias	0.0065	0.0023	0.0062	0.0047	0.0064	0.0052	0.0042	-0.0016	0.0057	-0.0029	0.0050	-0.0018
mse	0.0019	0.0020	0.0028	0.0035	0.0025	0.0029	0.0011	0.0016	0.0018	0.0027	0.0016	0.0024
var	0.0018	0.0020	0.0028	0.0034	0.0025	0.0029	0.0011	0.0016	0.0018	0.0026	0.0016	0.0024
	BALM		BAMM		BALTS		BALM		BAMM		BALTS	
mean	0.1990	0.1968	0.2026	0.2014	0.2043	0.2021	0.2001	0.1946	0.2040	0.1951	0.2030	0.1966
bias	-0.0010	-0.0032	0.0026	0.0014	0.0043	0.0021	0.0001	-0.0054	0.0040	-0.0049	0.0030	-0.0034
mse	0.0018	0.0020	0.0028	0.0034	0.0025	0.0029	0.0011	0.0016	0.0018	0.0027	0.0015	0.0024
var	0.0018	0.0020	0.0028	0.0034	0.0025	0.0029	0.0011	0.0015	0.0018	0.0026	0.0015	0.0024
	GPHTLM		GPHTMM		GPHTLTS		GPHTLM		GPHTMM		GPHTLTS	
mean	0.2348	0.2341	0.2274	0.2204	0.2320	0.2292	0.2188	0.2188	0.2121	0.2003	0.2165	0.2119
bias	0.0348	0.0341	0.0274	0.0204	0.0320	0.0292	0.0188	0.0188	0.0121	0.0003	0.0165	0.0119
mse	0.0062	0.0070	0.0088	0.0093	0.0065	0.0067	0.0033	0.0045	0.0055	0.0083	0.0035	0.0046
var	0.0050	0.0058	0.0081	0.0089	0.0055	0.0059	0.0030	0.0041	0.0054	0.0083	0.0032	0.0045

Table 1.7: Semiparametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ processes, when $p=P=0=q=Q$, $d=0.2=D$, $s=12$, $n\in\{1000,2000\}$, $\ell=2$ and $\alpha=0.8$.

	$n=1000$						$n=2000$					
	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$
	GPHLS		GPHMM		GPHLTS		GPHLS		GPHMM		GPHLTS	
mean	0.2289	0.2237	0.2205	0.2069	0.2272	0.2177	0.2149	0.2138	0.2112	0.2026	0.2155	0.2107
bias	0.0289	0.0237	0.0205	0.0069	0.0272	0.0177	0.0149	0.0138	0.0112	0.0026	0.0155	0.0107
mse	0.0030	0.0035	0.0050	0.0061	0.0032	0.0037	0.0015	0.0018	0.0026	0.0036	0.0017	0.0020
var	0.0022	0.0029	0.0046	0.0061	0.0025	0.0034	0.0013	0.0017	0.0025	0.0036	0.0014	0.0019
	RLM		RMM		RLTS		RLM		RMM		RLTS	
mean	0.2218	0.2185	0.2184	0.2048	0.2222	0.2136	0.2112	0.2109	0.2090	0.2013	0.2126	0.2088
bias	0.0218	0.0185	0.0184	0.0048	0.0222	0.0136	0.0112	0.0109	0.0090	0.0013	0.0126	0.0088
mse	0.0025	0.0032	0.0047	0.0060	0.0028	0.0034	0.0013	0.0017	0.0026	0.0037	0.0016	0.0020
var	0.0021	0.0028	0.0044	0.0060	0.0023	0.0033	0.0012	0.0016	0.0025	0.0037	0.0014	0.0019
	SRLM		SRMM		SRLTS		SRLM		SRMM		SRLTS	
mean	0.2045	0.2034	0.2037	0.2039	0.2049	0.2047	0.2029	0.2011	0.2032	0.2001	0.2025	0.2022
bias	0.0045	0.0034	0.0037	0.0039	0.0049	0.0047	0.0029	0.0011	0.0032	0.0001	0.0025	0.0022
mse	0.0012	0.0015	0.0019	0.0024	0.0016	0.0023	0.0008	0.0009	0.0012	0.0015	0.0011	0.0013
var	0.0012	0.0015	0.0019	0.0024	0.0016	0.0022	0.0008	0.0009	0.0012	0.0015	0.0010	0.0013
	BALM		BAMM		BALTS		BALM		BAMM		BALTS	
mean	0.1990	0.1994	0.2021	0.2021	0.2025	0.2035	0.1994	0.1984	0.2018	0.1987	0.2020	0.2009
bias	-0.0010	-0.0006	0.0021	0.0021	0.0025	0.0035	-0.0006	-0.0016	0.0018	-0.0013	0.0020	0.0009
mse	0.0012	0.0015	0.0019	0.0024	0.0016	0.0022	0.0008	0.0009	0.0012	0.0016	0.0010	0.0013
var	0.0012	0.0015	0.0019	0.0024	0.0016	0.0022	0.0008	0.0009	0.0012	0.0016	0.0010	0.0013
	GPHTLM		GPHTMM		GPHTLTS		GPHTLM		GPHTMM		GPHTLTS	
mean	0.2290	0.2254	0.2205	0.2128	0.2268	0.2208	0.2148	0.2170	0.2079	0.2053	0.2131	0.2124
bias	0.0290	0.0254	0.0205	0.0128	0.0268	0.0208	0.0148	0.0170	0.0079	0.0053	0.0131	0.0124
mse	0.0042	0.0048	0.0061	0.0071	0.0046	0.0049	0.0021	0.0027	0.0037	0.0050	0.0023	0.0030
var	0.0034	0.0041	0.0057	0.0070	0.0039	0.0045	0.0019	0.0024	0.0037	0.0050	0.0021	0.0028

Table 1.8: Semiparametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ processes, when $p=P=0=q=Q$, $d=0.2=D$, $s=12$, $n\in\{1000,2000\}$, $\ell=2$ and $\alpha=0.89$.

	$n=1000$						$n=2000$					
	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$	$\widehat{d}$	$\widehat{D}$
	GPHLS		GPHMM		GPHLTS		GPHLS		GPHMM		GPHLTS	
mean	0.2199	0.2204	0.2103	0.2081	0.2167	0.2159	0.2111	0.2116	0.2073	0.2034	0.2103	0.2100
bias	0.0199	0.0204	0.0103	0.0081	0.0167	0.0159	0.0111	0.0116	0.0073	0.0034	0.0103	0.0100
mse	0.0016	0.0020	0.0032	0.0041	0.0019	0.0022	0.0008	0.0008	0.0016	0.0018	0.0010	0.0011
var	0.0012	0.0016	0.0031	0.0041	0.0016	0.0020	0.0007	0.0007	0.0015	0.0018	0.0009	0.0010
	RLM		RMM		RLTS		RLM		RMM		RLTS	
mean	0.2147	0.2172	0.2090	0.2083	0.2134	0.2141	0.2085	0.2100	0.2061	0.2026	0.2093	0.2082
bias	0.0147	0.0172	0.0090	0.0083	0.0134	0.0141	0.0085	0.0100	0.0061	0.0026	0.0093	0.0082
mse	0.0014	0.0019	0.0031	0.0041	0.0017	0.0022	0.0007	0.0008	0.0015	0.0018	0.0009	0.0010
var	0.0012	0.0016	0.0030	0.0040	0.0015	0.0020	0.0006	0.0007	0.0015	0.0018	0.0008	0.0010
	SRLM		SRMM		SRLTS		SRLM		SRMM		SRLTS	
mean	0.2038	0.2024	0.2036	0.2047	0.2040	0.2042	0.2020	0.2027	0.2014	0.2033	0.2021	0.2036
bias	0.0038	0.0024	0.0036	0.0047	0.0040	0.0042	0.0020	0.0027	0.0014	0.0033	0.0021	0.0036
mse	0.0007	0.0009	0.0012	0.0016	0.0010	0.0014	0.0004	0.0004	0.0007	0.0007	0.0005	0.0007
var	0.0007	0.0009	0.0012	0.0016	0.0010	0.0014	0.0004	0.0004	0.0007	0.0007	0.0005	0.0006
	BALM		BAMM		BALTS		BALM		BAMM		BALTS	
mean	0.2001	0.2001	0.2022	0.2029	0.2017	0.2038	0.1997	0.2012	0.2003	0.2026	0.2012	0.2029
bias	0.0001	0.0001	0.0022	0.0029	0.0017	0.0038	-0.0003	0.0012	0.0003	0.0026	0.0012	0.0029
mse	0.0007	0.0009	0.0011	0.0016	0.0010	0.0014	0.0004	0.0004	0.0006	0.0007	0.0005	0.0007
var	0.0007	0.0009	0.0011	0.0016	0.0010	0.0014	0.0004	0.0004	0.0006	0.0007	0.0005	0.0006
	GPHTLM		GPHTMM		GPHTLTS		GPHTLM		GPHTMM		GPHTLTS	
mean	0.2183	0.2229	0.2102	0.2115	0.2154	0.2207	0.2109	0.2142	0.2052	0.2082	0.2085	0.2126
bias	0.0183	0.0229	0.0102	0.0115	0.0154	0.0207	0.0109	0.0142	0.0052	0.0082	0.0085	0.0126
mse	0.0023	0.0030	0.0037	0.0049	0.0024	0.0034	0.0011	0.0012	0.0019	0.0024	0.0013	0.0016
var	0.0020	0.0024	0.0036	0.0047	0.0022	0.0029	0.0010	0.0010	0.0018	0.0024	0.0012	0.0014

Table 1.9: Parametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ process, when $p=1$, $P=0=q=Q$, $d=0.2=D$, $\phi_1=0.5$, $s\in\{4,6,12\}$ and $n\in\{1000,2000\}$.

	$s=4$																	
	$n=1000$									$n=2000$								
	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2704	0.1843	0.4398	0.2086	0.1972	0.4884	0.1801	0.1989	0.5148	0.2586	0.1897	0.4479	0.2086	0.1972	0.4884	0.1828	0.1992	0.5145
bias	0.0704	-0.0157	-0.0602	0.0086	-0.0028	-0.0116	-0.0199	-0.0011	0.0148	0.0586	-0.0103	-0.0521	0.0086	-0.0028	-0.0116	-0.0172	-0.0008	0.0145
mse	0.0207	0.0012	0.0191	0.0064	0.0009	0.0082	0.0071	0.0008	0.0076	0.0093	0.0006	0.0088	0.0064	0.0009	0.0082	0.0051	0.0004	0.0056
var	0.0157	0.0010	0.0155	0.0063	0.0009	0.0080	0.0067	0.0008	0.0074	0.0058	0.0005	0.0061	0.0063	0.0009	0.0080	0.0049	0.0004	0.0054
	$s=6$																	
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2790	0.1865	0.4318	0.2112	0.1980	0.4873	0.1744	0.2020	0.5174	0.2563	0.1922	0.4498	0.2069	0.1985	0.4914	0.1803	0.2004	0.5163
bias	0.0790	-0.0135	-0.0682	0.0112	-0.0020	-0.0127	-0.0256	0.0020	0.0174	0.0563	-0.0078	-0.0502	0.0069	-0.0015	-0.0086	-0.0197	0.0004	0.0163
mse	0.0161	0.0011	0.0148	0.0086	0.0013	0.0106	0.0081	0.0007	0.0122	0.0080	0.0005	0.0078	0.0056	0.0008	0.0073	0.0039	0.0004	0.0043
var	0.0099	0.0010	0.0102	0.0085	0.0013	0.0104	0.0075	0.0007	0.0119	0.0048	0.0005	0.0053	0.0055	0.0008	0.0073	0.0035	0.0004	0.0041
	$s=12$																	
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2804	0.1826	0.4292	0.2118	0.1990	0.4857	0.1756	0.2020	0.5171	0.2525	0.1922	0.4530	0.2038	0.2012	0.4958	0.1800	0.2021	0.5176
bias	0.0804	-0.0174	-0.0708	0.0118	-0.0010	-0.0143	-0.0244	0.0020	0.0171	0.0525	-0.0078	-0.0470	0.0038	0.0012	-0.0042	-0.0200	0.0021	0.0176
mse	0.0144	0.0013	0.0135	0.0070	0.0011	0.0089	0.0064	0.0008	0.0073	0.0073	0.0005	0.0070	0.0044	0.0007	0.0056	0.0039	0.0003	0.0044
var	0.0079	0.0010	0.0085	0.0069	0.0011	0.0087	0.0058	0.0008	0.0070	0.0046	0.0004	0.0048	0.0044	0.0007	0.0056	0.0035	0.0003	0.0041

Table 1.10: Parametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ process, when $p=1$, $P=0=q=Q$, $d=0.2=D$, $\phi_1=-0.5$, $s\in\{4,6,12\}$ and $n\in\{1000,2000\}$.

	$s=4$																	
	$n=1000$									$n=2000$								
	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2319	0.1980	-0.5066	0.2045	0.2000	-0.5019	0.1947	0.1979	-0.4972	0.2217	0.1998	-0.5044	0.2040	0.2003	-0.5000	0.1985	0.1993	-0.4986
bias	0.0319	-0.0020	-0.0066	0.0045	0.0000	-0.0019	-0.0053	-0.0021	0.0028	0.0217	-0.0002	-0.0044	0.0040	0.0003	0.0000	-0.0015	-0.0007	0.0014
mse	0.0025	0.0010	0.0013	0.0013	0.0009	0.0015	0.0012	0.0008	0.0012	0.0013	0.0004	0.0006	0.0009	0.0006	0.0009	0.0006	0.0004	0.0006
var	0.0015	0.0010	0.0012	0.0013	0.0009	0.0015	0.0011	0.0008	0.0012	0.0008	0.0004	0.0006	0.0008	0.0006	0.0009	0.0006	0.0004	0.0006
	$s=6$																	
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2321	0.1972	-0.5035	0.2050	0.2011	-0.4974	0.1941	0.1997	-0.4942	0.2201	0.1997	-0.5037	0.2031	0.2007	-0.5003	0.1967	0.1998	-0.4974
bias	0.0321	-0.0028	-0.0035	0.0050	0.0011	0.0026	-0.0059	-0.0003	0.0058	0.0201	-0.0003	-0.0037	0.0031	0.0007	-0.0003	-0.0033	-0.0002	0.0026
mse	0.0026	0.0010	0.0012	0.0013	0.0009	0.0014	0.0011	0.0007	0.0012	0.0011	0.0005	0.0006	0.0007	0.0005	0.0008	0.0005	0.0003	0.0006
var	0.0016	0.0010	0.0012	0.0013	0.0009	0.0014	0.0011	0.0007	0.0011	0.0007	0.0005	0.0006	0.0007	0.0005	0.0008	0.0005	0.0003	0.0006
	$s=12$																	
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2309	0.1880	-0.5030	0.2028	0.1994	-0.4967	0.1918	0.2006	-0.4926	0.2208	0.1964	-0.5048	0.2027	0.2008	-0.5010	0.1968	0.2005	-0.4977
bias	0.0309	-0.0120	-0.0030	0.0028	-0.0006	0.0033	-0.0082	0.0006	0.0074	0.0208	-0.0036	-0.0048	0.0027	0.0008	-0.0010	-0.0032	0.0005	0.0023
mse	0.0026	0.0012	0.0012	0.0012	0.0008	0.0013	0.0010	0.0007	0.0012	0.0012	0.0005	0.0006	0.0007	0.0005	0.0009	0.0004	0.0003	0.0006
var	0.0017	0.0011	0.0012	0.0012	0.0008	0.0013	0.0010	0.0007	0.0011	0.0008	0.0005	0.0006	0.0007	0.0005	0.0009	0.0004	0.0003	0.0006

Table 1.11: Parametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ process, when $p=0=q=Q$, $P=1$, $d=0.2=D$, $\Phi_1=0.5$, $s\in\{4,6,12\}$ and $n\in\{1000,2000\}$.

	$s=4$																	
	$n=1000$									$n=2000$								
	$\widehat{d}$	$\widehat{D}$	$\widehat{\Phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\Phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\Phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\Phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\Phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\Phi_1}$
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2367	0.1942	0.4973	0.2064	0.1823	0.5077	0.1967	0.1822	0.5119	0.2193	0.1926	0.5031	0.2040	0.1982	0.4980	0.1984	0.1904	0.5066
bias	0.0367	-0.0058	-0.0027	0.0064	-0.0177	0.0077	-0.0033	-0.0178	0.0119	0.0193	-0.0074	0.0031	0.0040	-0.0018	-0.0020	-0.0016	-0.0096	0.0066
mse	0.0022	0.0104	0.0107	0.0011	0.0111	0.0124	0.0007	0.0056	0.0062	0.0008	0.0059	0.0061	0.0007	0.0046	0.0060	0.0003	0.0031	0.0036
var	0.0009	0.0103	0.0107	0.0011	0.0108	0.0124	0.0007	0.0053	0.0061	0.0004	0.0059	0.0061	0.0007	0.0046	0.0060	0.0003	0.0030	0.0036
	$s=6$																	
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2385	0.1800	0.5132	0.2047	0.1944	0.5014	0.1970	0.1906	0.5037	0.2207	0.1807	0.5155	0.2047	0.1944	0.5014	0.1983	0.1949	0.4969
bias	0.0385	-0.0200	0.0132	0.0047	-0.0056	0.0014	-0.0030	-0.0094	0.0037	0.0207	-0.0193	0.0155	0.0047	-0.0056	0.0014	-0.0017	-0.0051	-0.0031
mse	0.0025	0.0108	0.0110	0.0007	0.0043	0.0056	0.0007	0.0056	0.0066	0.0009	0.0067	0.0068	0.0007	0.0043	0.0056	0.0004	0.0041	0.0117
var	0.0011	0.0104	0.0108	0.0006	0.0043	0.0056	0.0007	0.0056	0.0066	0.0005	0.0064	0.0066	0.0006	0.0043	0.0056	0.0004	0.0041	0.0117
	$s=12$																	
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2390	0.1149	0.5747	0.2085	0.1848	0.5053	0.1972	0.2031	0.4906	0.2244	0.1372	0.5591	0.2056	0.1893	0.5051	0.1990	0.1973	0.5000
bias	0.0390	-0.0851	0.0747	0.0085	-0.0152	0.0053	-0.0028	0.0031	-0.0094	0.0244	-0.0628	0.0591	0.0056	-0.0107	0.0051	-0.0010	-0.0027	0.0000
mse	0.0028	0.0175	0.0161	0.0012	0.0070	0.0085	0.0008	0.0061	0.0112	0.0011	0.0103	0.0098	0.0007	0.0046	0.0058	0.0003	0.0030	0.0034
var	0.0012	0.0102	0.0106	0.0011	0.0067	0.0085	0.0008	0.0061	0.0111	0.0005	0.0064	0.0063	0.0006	0.0045	0.0058	0.0003	0.0030	0.0034

Table 1.12: Parametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ process, when $p=0=q=Q$, $P=1$, $d=0.2=D$, $\Phi_1=-0.5$, $s\in\{4,6,12\}$ and $n\in\{1000,2000\}$.

	$s=4$																	
	$n=1000$									$n=2000$								
	$\widehat{d}$	$\widehat{D}$	$\widehat{\Phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\Phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\Phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\Phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\Phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\Phi_1}$
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2245	0.1982	-0.4949	0.2026	0.1998	-0.4948	0.2384	0.1989	-0.4955	0.2176	0.2012	-0.4998	0.2036	0.1994	-0.4985	0.1989	0.1975	-0.4983
bias	0.0245	-0.0018	0.0051	0.0026	-0.0002	0.0052	0.0384	-0.0011	0.0045	0.0176	0.0012	0.0002	0.0036	-0.0006	0.0015	-0.0011	-0.0025	0.0017
mse	0.0017	0.0014	0.0013	0.0009	0.0011	0.0014	0.0522	0.0041	0.0012	0.0008	0.0006	0.0006	0.0005	0.0007	0.0008	0.0003	0.0005	0.0005
var	0.0011	0.0014	0.0013	0.0009	0.0011	0.0014	0.0508	0.0041	0.0011	0.0005	0.0006	0.0006	0.0005	0.0007	0.0008	0.0003	0.0005	0.0005
	$s=6$																	
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2251	0.1792	-0.4785	0.2038	0.1977	-0.4921	0.1957	0.2008	-0.5005	0.2150	0.1996	-0.4968	0.2014	0.2001	-0.4973	0.1973	0.1984	-0.4981
bias	0.0251	-0.0208	0.0215	0.0038	-0.0023	0.0079	-0.0043	0.0008	-0.0005	0.0150	-0.0004	0.0032	0.0014	0.0001	0.0027	-0.0027	-0.0016	0.0019
mse	0.0018	0.0023	0.0018	0.0008	0.0013	0.0014	0.0007	0.0010	0.0011	0.0007	0.0007	0.0006	0.0005	0.0007	0.0008	0.0003	0.0005	0.0006
var	0.0012	0.0018	0.0014	0.0008	0.0013	0.0014	0.0007	0.0010	0.0011	0.0004	0.0007	0.0006	0.0005	0.0007	0.0008	0.0003	0.0005	0.0006
	$s=12$																	
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2263	0.1765	-0.4747	0.2044	0.1967	-0.4896	0.1961	0.1984	-0.4969	0.2158	0.1935	-0.4912	0.2020	0.1993	-0.4955	0.1961	0.1984	-0.4969
bias	0.0263	-0.0235	0.0253	0.0044	-0.0033	0.0104	-0.0039	-0.0016	0.0031	0.0158	-0.0065	0.0088	0.0020	-0.0007	0.0045	-0.0039	-0.0016	0.0031
mse	0.0018	0.0023	0.0019	0.0008	0.0011	0.0014	0.0007	0.0009	0.0011	0.0008	0.0007	0.0007	0.0005	0.0007	0.0009	0.0007	0.0009	0.0011
var	0.0011	0.0017	0.0012	0.0008	0.0011	0.0013	0.0007	0.0009	0.0011	0.0005	0.0007	0.0006	0.0005	0.0007	0.0009	0.0007	0.0009	0.0011

Table 1.13: Parametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ process, when $p=P=0=Q$, $q=1$, $d=0.2=D$, $\theta_1=0.5$, $s\in\{4,6,12\}$ and $n\in\{1000,2000\}$.

	$s=4$																	
	$n=1000$									$n=2000$								
	$\widehat{d}$	$\widehat{D}$	$\widehat{\theta}_1$	$\widehat{d}$	$\widehat{D}$	$\widehat{\theta}_1$	$\widehat{d}$	$\widehat{D}$	$\widehat{\theta}_1$	$\widehat{d}$	$\widehat{D}$	$\widehat{\theta}_1$	$\widehat{d}$	$\widehat{D}$	$\widehat{\theta}_1$	$\widehat{d}$	$\widehat{D}$	$\widehat{\theta}_1$
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2904	0.1908	0.5697	0.2165	0.2003	0.5101	0.1797	0.2018	0.4789	0.2738	0.1950	0.5617	0.2116	0.2021	0.5081	0.1859	0.2028	0.4849
bias	0.0904	-0.0092	0.0697	0.0165	0.0003	0.0101	-0.0203	0.0018	-0.0211	0.0738	-0.0050	0.0617	0.0116	0.0021	0.0081	-0.0141	0.0028	-0.0151
mse	0.0194	0.0010	0.0152	0.0098	0.0010	0.0104	0.0074	0.0008	0.0083	0.0125	0.0004	0.0106	0.0060	0.0006	0.0068	0.0039	0.0004	0.0045
var	0.0113	0.0009	0.0103	0.0095	0.0010	0.0103	0.0070	0.0008	0.0079	0.0071	0.0004	0.0068	0.0059	0.0006	0.0067	0.0037	0.0004	0.0043
	$s=6$																	
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2898	0.1886	0.5698	0.2099	0.2014	0.5036	0.1720	0.2040	0.4704	0.2763	0.1915	0.5661	0.2116	0.1999	0.5077	0.1874	0.2007	0.4876
bias	0.0898	-0.0114	0.0698	0.0099	0.0014	0.0036	-0.0280	0.0040	-0.0296	0.0763	-0.0085	0.0661	0.0116	-0.0001	0.0077	-0.0126	0.0007	-0.0124
mse	0.0200	0.0011	0.0161	0.0082	0.0010	0.0095	0.0073	0.0008	0.0086	0.0129	0.0005	0.0111	0.0056	0.0005	0.0067	0.0037	0.0003	0.0043
var	0.0120	0.0009	0.0113	0.0082	0.0010	0.0095	0.0065	0.0008	0.0077	0.0071	0.0004	0.0068	0.0055	0.0005	0.0067	0.0035	0.0003	0.0041
	$s=12$																	
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2842	0.1808	0.5673	0.2089	0.1990	0.5052	0.1728	0.2018	0.4723	0.2711	0.1922	0.5603	0.2111	0.1998	0.5101	0.1856	0.2027	0.4837
bias	0.0842	-0.0192	0.0673	0.0089	-0.0010	0.0052	-0.0272	0.0018	-0.0277	0.0711	-0.0078	0.0603	0.0111	-0.0002	0.0101	-0.0144	0.0027	-0.0163
mse	0.0182	0.0013	0.0148	0.0077	0.0008	0.0084	0.0071	0.0007	0.0078	0.0114	0.0005	0.0099	0.0053	0.0005	0.0062	0.0031	0.0004	0.0037
var	0.0111	0.0009	0.0102	0.0076	0.0008	0.0083	0.0064	0.0007	0.0071	0.0064	0.0004	0.0062	0.0052	0.0005	0.0061	0.0029	0.0003	0.0035

Table 1.14: Parametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ process, when $p=P=0=Q$, $q=1$, $d=0.2=D$, $\theta_1=-0.5$, $s\in\{4,6,12\}$ and $n\in\{1000,2000\}$.

	$s=4$																	
	$n=1000$									$n=2000$								
	$\widehat{d}$	$\widehat{D}$	$\widehat{\theta}_1$	$\widehat{d}$	$\widehat{D}$	$\widehat{\theta}_1$	$\widehat{d}$	$\widehat{D}$	$\widehat{\theta}_1$	$\widehat{d}$	$\widehat{D}$	$\widehat{\theta}_1$	$\widehat{d}$	$\widehat{D}$	$\widehat{\theta}_1$	$\widehat{d}$	$\widehat{D}$	$\widehat{\theta}_1$
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2319	0.1966	-0.4903	0.2060	0.2000	-0.4971	0.1927	0.2003	-0.5036	0.2200	0.2006	-0.4926	0.2036	0.2007	-0.4971	0.1981	0.1999	-0.4986
bias	0.0319	-0.0034	0.0097	0.0060	0.0000	0.0029	-0.0073	0.0003	-0.0036	0.0200	0.0006	0.0074	0.0036	0.0007	0.0029	-0.0019	-0.0001	0.0014
mse	0.0026	0.0010	0.0015	0.0015	0.0011	0.0020	0.0011	0.0008	0.0012	0.0011	0.0004	0.0007	0.0009	0.0006	0.0014	0.0007	0.0004	0.0016
var	0.0016	0.0010	0.0014	0.0014	0.0011	0.0020	0.0011	0.0008	0.0012	0.0007	0.0004	0.0006	0.0009	0.0006	0.0014	0.0007	0.0004	0.0016
	$s=6$																	
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2319	0.1969	-0.4907	0.2060	0.2006	-0.4981	0.1953	0.1997	-0.5002	0.2209	0.2007	-0.4934	0.2037	0.2012	-0.4974	0.1997	0.2006	-0.4998
bias	0.0319	-0.0031	0.0093	0.0060	0.0006	0.0019	-0.0047	-0.0003	-0.0002	0.0209	0.0007	0.0066	0.0037	0.0012	0.0026	-0.0003	0.0006	0.0002
mse	0.0026	0.0009	0.0014	0.0014	0.0010	0.0020	0.0013	0.0007	0.0039	0.0011	0.0004	0.0007	0.0008	0.0006	0.0013	0.0005	0.0004	0.0007
var	0.0016	0.0009	0.0013	0.0014	0.0010	0.0020	0.0013	0.0007	0.0039	0.0007	0.0004	0.0006	0.0008	0.0006	0.0013	0.0005	0.0004	0.0007
	$s=12$																	
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2318	0.1870	-0.4903	0.2058	0.1988	-0.4982	0.1928	0.1998	-0.5049	0.2191	0.1962	-0.4933	0.2020	0.2003	-0.4991	0.1969	0.2005	-0.5004
bias	0.0318	-0.0130	0.0097	0.0058	-0.0012	0.0018	-0.0072	-0.0002	-0.0049	0.0191	-0.0038	0.0067	0.0020	0.0003	0.0009	-0.0031	0.0005	-0.0004
mse	0.0027	0.0011	0.0016	0.0013	0.0009	0.0019	0.0010	0.0007	0.0012	0.0010	0.0005	0.0007	0.0007	0.0006	0.0013	0.0007	0.0004	0.0022
var	0.0017	0.0010	0.0015	0.0013	0.0009	0.0019	0.0009	0.0007	0.0011	0.0007	0.0005	0.0007	0.0007	0.0006	0.0013	0.0007	0.0004	0.0022

Table 1.15: Parametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ process, when $p=P=0=q$, $Q=1$, $d=0.2=D$, $\Theta_1=0.5$, $s\in\{4,6,12\}$ and $n\in\{1000,2000\}$.

	$s=4$																	
	$n=1000$									$n=2000$								
	$\widehat{d}$	$\widehat{D}$	$\widehat{\Theta_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\Theta_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\Theta_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\Theta_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\Theta_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\Theta_1}$
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2214	0.1872	0.4785	0.2017	0.2020	0.4976	0.1949	0.1906	0.4895	0.2128	0.2070	0.5018	0.2012	0.2038	0.5010	0.1969	0.1938	0.4924
bias	0.0214	-0.0128	-0.0215	0.0017	0.0020	-0.0024	-0.0051	-0.0094	-0.0105	0.0128	0.0070	0.0018	0.0012	0.0038	0.0010	-0.0031	-0.0062	-0.0076
mse	0.0014	0.0105	0.0106	0.0009	0.0071	0.0084	0.0008	0.0061	0.0073	0.0006	0.0064	0.0063	0.0005	0.0041	0.0049	0.0004	0.0030	0.0036
var	0.0010	0.0104	0.0101	0.0009	0.0071	0.0084	0.0007	0.0060	0.0072	0.0005	0.0063	0.0063	0.0005	0.0041	0.0049	0.0004	0.0029	0.0035
	$s=6$																	
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2232	0.1567	0.4450	0.2034	0.1963	0.4883	0.1964	0.1944	0.4923	0.2135	0.1864	0.4814	0.2016	0.2026	0.5000	0.1978	0.1985	0.4985
bias	0.0232	-0.0433	-0.0550	0.0034	-0.0037	-0.0117	-0.0036	-0.0056	-0.0077	0.0135	-0.0136	-0.0186	0.0016	0.0026	0.0000	-0.0022	-0.0015	-0.0015
mse	0.0016	0.0093	0.0106	0.0008	0.0065	0.0077	0.0007	0.0064	0.0074	0.0007	0.0055	0.0057	0.0005	0.0043	0.0053	0.0004	0.0030	0.0035
var	0.0010	0.0075	0.0076	0.0008	0.0065	0.0076	0.0007	0.0063	0.0074	0.0005	0.0053	0.0053	0.0005	0.0043	0.0053	0.0004	0.0030	0.0035
	$s=12$																	
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2252	0.0898	0.3726	0.2030	0.1840	0.4742	0.1952	0.2081	0.5061	0.2135	0.1411	0.4313	0.2010	0.1963	0.4898	0.1972	0.2064	0.5047
bias	0.0252	-0.1102	-0.1274	0.0030	-0.0160	-0.0258	-0.0048	0.0081	0.0061	0.0135	-0.0589	-0.0687	0.0010	-0.0037	-0.0102	-0.0028	0.0064	0.0047
mse	0.0019	0.0158	0.0199	0.0008	0.0065	0.0081	0.0007	0.0078	0.0090	0.0007	0.0071	0.0084	0.0005	0.0038	0.0046	0.0003	0.0035	0.0040
var	0.0013	0.0037	0.0037	0.0008	0.0063	0.0075	0.0006	0.0077	0.0089	0.0005	0.0037	0.0037	0.0005	0.0038	0.0045	0.0003	0.0034	0.0040

Table 1.16: Parametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ process, when $p=P=0=q$, $Q=1$, $d=0.2=D$, $\Theta_1=-0.5$, $s\in\{4,6,12\}$ and $n\in\{1000,2000\}$.

	$s=4$																	
	$n=1000$									$n=2000$								
	$\widehat{d}$	$\widehat{D}$	$\widehat{\Theta_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\Theta_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\Theta_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\Theta_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\Theta_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\Theta_1}$
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2314	0.1994	-0.4913	0.2048	0.2017	-0.4923	0.1977	0.2011	-0.4862	0.2198	0.2013	-0.4959	0.2033	0.2003	-0.4977	0.1993	0.1987	-0.4984
bias	0.0314	-0.0006	0.0087	0.0048	0.0017	0.0077	-0.0023	0.0011	0.0138	0.0198	0.0013	0.0041	0.0033	0.0003	0.0023	-0.0007	-0.0013	0.0016
mse	0.0020	0.0014	0.0013	0.0010	0.0014	0.0023	0.0009	0.0021	0.0142	0.0009	0.0006	0.0006	0.0006	0.0008	0.0015	0.0004	0.0007	0.0019
var	0.0010	0.0014	0.0013	0.0010	0.0014	0.0022	0.0009	0.0021	0.0140	0.0005	0.0006	0.0006	0.0006	0.0008	0.0015	0.0004	0.0007	0.0019
	$s=6$																	
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2330	0.1956	-0.4962	0.2068	0.2005	-0.4958	0.1973	0.1987	-0.4982	0.2207	0.2020	-0.4945	0.2041	0.2024	-0.4951	0.1994	0.2006	-0.4964
bias	0.0330	-0.0044	0.0038	0.0068	0.0005	0.0042	-0.0027	-0.0013	0.0018	0.0207	0.0020	0.0055	0.0041	0.0024	0.0049	-0.0006	0.0006	0.0036
mse	0.0021	0.0014	0.0013	0.0010	0.0012	0.0021	0.0008	0.0010	0.0019	0.0009	0.0006	0.0007	0.0006	0.0008	0.0015	0.0003	0.0006	0.0020
var	0.0010	0.0014	0.0012	0.0009	0.0012	0.0021	0.0007	0.0010	0.0019	0.0005	0.0006	0.0007	0.0006	0.0008	0.0015	0.0003	0.0006	0.0019
	$s=12$																	
	W			W-MCMC			BR			W			W-MCMC			BR		
mean	0.2292	0.1810	-0.4988	0.2049	0.2003	-0.4873	0.1954	0.2023	-0.4953	0.2198	0.1942	-0.4974	0.2039	0.2023	-0.4934	0.1969	0.2033	-0.4948
bias	0.0292	-0.0190	0.0012	0.0049	0.0003	0.0127	-0.0046	0.0023	0.0047	0.0198	-0.0058	0.0026	0.0039	0.0023	0.0066	-0.0031	0.0033	0.0052
mse	0.0020	0.0021	0.0012	0.0010	0.0014	0.0022	0.0007	0.0010	0.0011	0.0008	0.0007	0.0006	0.0006	0.0007	0.0014	0.0003	0.0007	0.0031
var	0.0011	0.0017	0.0012	0.0010	0.0014	0.0020	0.0007	0.0010	0.0011	0.0005	0.0007	0.0006	0.0005	0.0007	0.0013	0.0003	0.0007	0.0031

Table 1.17: Parametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ process, when $p = P = 1 = Q$, $q = 0$, $d = 0.2 = D$, $\Phi_1 = -0.5$, $\phi_1 = 0.5 = \Theta_1$, $s \in \{4, 6, 12\}$ and $n = 1000$.

	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\Phi_1}$	$\widehat{\Theta_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\Phi_1}$	$\widehat{\Theta_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\Phi_1}$	$\widehat{\Theta_1}$
$s = 4$															
	W					W-MCMC					BR				
mean	0.2915	0.1534	0.4078	-0.5001	0.4567	0.2161	0.2007	0.4790	-0.5008	0.4971	0.1788	0.1888	0.5138	-0.5021	0.4862
bias	0.0915	-0.0466	-0.0922	-0.0001	-0.0433	0.0161	0.0007	-0.0210	-0.0008	-0.0029	-0.0212	-0.0112	0.0138	-0.0021	-0.0138
mse	0.0167	0.0104	0.0171	0.0013	0.0113	0.0089	0.0097	0.0100	0.0018	0.0125	0.0079	0.0061	0.0089	0.0013	0.0077
var	0.0084	0.0083	0.0086	0.0013	0.0094	0.0086	0.0097	0.0096	0.0018	0.0125	0.0074	0.0060	0.0087	0.0013	0.0075
$s = 6$															
	W					W-MCMC					BR				
mean	0.2765	0.1363	0.4230	-0.4998	0.4300	0.2168	0.1915	0.4793	-0.4998	0.4843	0.1734	0.1885	0.5173	-0.4992	0.4846
bias	0.0765	-0.0637	-0.0770	0.0002	-0.0700	0.0168	-0.0085	-0.0207	0.0002	-0.0157	-0.0266	-0.0115	0.0173	0.0008	-0.0154
mse	0.0148	0.0098	0.0148	0.0013	0.0118	0.0077	0.0088	0.0090	0.0019	0.0123	0.0072	0.0065	0.0092	0.0017	0.0083
var	0.0090	0.0058	0.0089	0.0013	0.0069	0.0074	0.0087	0.0086	0.0019	0.0120	0.0065	0.0063	0.0089	0.0017	0.0080
$s = 12$															
	W					W-MCMC					BR				
mean	0.2610	0.0763	0.4395	-0.4969	0.3540	0.2068	0.1752	0.4886	-0.4975	0.4557	0.1655	0.1947	0.5270	-0.5012	0.4855
bias	0.0610	-0.1237	-0.0605	0.0031	-0.1460	0.0068	-0.0248	-0.0114	0.0025	-0.0443	-0.0345	-0.0053	0.0270	-0.0012	-0.0145
mse	0.0140	0.0182	0.0142	0.0013	0.0256	0.0070	0.0090	0.0088	0.0020	0.0142	0.0073	0.0065	0.0077	0.0015	0.0085
var	0.0103	0.0029	0.0106	0.0013	0.0043	0.0069	0.0083	0.0087	0.0020	0.0122	0.0061	0.0064	0.0070	0.0015	0.0083

Table 1.18: Parametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ process, when $p = P = 1 = Q$, $q = 0$, $d = 0.2 = D$, $\Phi_1 = -0.5$, $\phi_1 = 0.5 = \Theta_1$, $s \in \{4, 6, 12\}$ and $n = 2000$.

	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\Phi_1}$	$\widehat{\Theta_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\Phi_1}$	$\widehat{\Theta_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\Phi_1}$	$\widehat{\Theta_1}$
$s = 4$															
	W					W-MCMC					BR				
mean	0.2632	0.1783	0.4375	-0.4992	0.4825	0.2135	0.1963	0.4849	-0.5024	0.4950	0.1829	0.1921	0.5133	-0.5000	0.4900
bias	0.0632	-0.0217	-0.0625	0.0008	-0.0175	0.0135	-0.0037	-0.0151	-0.0024	-0.0050	-0.0171	-0.0079	0.0133	0.0000	-0.0100
mse	0.0083	0.0052	0.0086	0.0006	0.0058	0.0064	0.0073	0.0073	0.0013	0.0099	0.0042	0.0029	0.0055	0.0009	0.0039
var	0.0043	0.0047	0.0047	0.0006	0.0055	0.0062	0.0073	0.0071	0.0013	0.0099	0.0039	0.0028	0.0054	0.0009	0.0038
$s = 6$															
	W					W-MCMC					BR				
mean	0.2501	0.1658	0.4489	-0.5002	0.4635	0.2028	0.1996	0.4940	-0.5011	0.4926	0.1774	0.1954	0.5190	-0.4996	0.4912
bias	0.0501	-0.0342	-0.0511	-0.0002	-0.0365	0.0028	-0.0004	-0.0060	-0.0011	-0.0074	-0.0226	-0.0046	0.0190	0.0004	-0.0088
mse	0.0071	0.0051	0.0075	0.0007	0.0060	0.0053	0.0080	0.0061	0.0013	0.0109	0.0046	0.0031	0.0049	0.0015	0.0039
var	0.0045	0.0040	0.0049	0.0007	0.0046	0.0053	0.0080	0.0061	0.0013	0.0109	0.0041	0.0030	0.0045	0.0015	0.0038
$s = 12$															
	W					W-MCMC					BR				
mean	0.2387	0.1241	0.4608	-0.5001	0.4121	0.1997	0.1965	0.4983	-0.5015	0.4858	0.1765	0.1979	0.5190	-0.4997	0.4930
bias	0.0387	-0.0759	-0.0392	-0.0001	-0.0879	-0.0003	-0.0035	-0.0017	-0.0015	-0.0142	-0.0235	-0.0021	0.0190	0.0003	-0.0070
mse	0.0068	0.0092	0.0069	0.0007	0.0118	0.0046	0.0084	0.0058	0.0014	0.0119	0.0043	0.0035	0.0046	0.0007	0.0045
var	0.0053	0.0034	0.0054	0.0007	0.0041	0.0046	0.0084	0.0058	0.0014	0.0117	0.0037	0.0035	0.0042	0.0007	0.0045

Table 1.19: Parametric estimation for the SARFIMA$(p,d,q) \times (P,D,Q)_s$ process, when $p = P = 1 = Q$, $q = 0$, $d = 0.2 = D$, $\phi_1 = -0.5 = \Phi_1$, $\Theta_1 = 0.5$, $s \in \{4, 6, 12\}$ and $n = 1000$.

	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\Phi_1}$	$\widehat{\Theta_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\Phi_1}$	$\widehat{\Theta_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\Phi_1}$	$\widehat{\Theta_1}$
$s = 4$															
	W					W-MCMC					BR				
mean	0.2252	0.1810	-0.5080	-0.5031	0.4705	0.2014	0.2000	-0.4972	-0.5001	0.4935	0.1930	0.1895	-0.4934	-0.5001	0.4863
bias	0.0252	-0.0190	-0.0080	-0.0031	-0.0295	0.0014	0.0000	0.0028	-0.0001	-0.0065	-0.0070	-0.0105	0.0066	-0.0001	-0.0137
mse	0.0022	0.0105	0.0014	0.0013	0.0112	0.0017	0.0079	0.0018	0.0018	0.0102	0.0012	0.0062	0.0015	0.0013	0.0077
var	0.0015	0.0101	0.0013	0.0013	0.0103	0.0017	0.0079	0.0018	0.0018	0.0102	0.0011	0.0061	0.0014	0.0013	0.0076
$s = 6$															
	W					W-MCMC					BR				
mean	0.2225	0.1547	-0.5079	-0.5025	0.4378	0.2023	0.1929	-0.4983	-0.4993	0.4829	0.1904	0.1886	-0.4937	-0.5020	0.4832
bias	0.0225	-0.0453	-0.0079	-0.0025	-0.0622	0.0023	-0.0071	0.0017	0.0007	-0.0171	-0.0096	-0.0114	0.0063	-0.0020	-0.0168
mse	0.0022	0.0095	0.0013	0.0013	0.0123	0.0015	0.0088	0.0017	0.0016	0.0117	0.0012	0.0063	0.0012	0.0013	0.0085
var	0.0016	0.0074	0.0012	0.0013	0.0084	0.0015	0.0088	0.0017	0.0016	0.0114	0.0011	0.0061	0.0011	0.0013	0.0082
$s = 12$															
	W					W-MCMC					BR				
mean	0.2266	0.0828	-0.5111	-0.4969	0.3592	0.1995	0.1791	-0.4980	-0.4932	0.4637	0.1905	0.1960	-0.4944	-0.4988	0.4914
bias	0.0266	-0.1172	-0.0111	0.0031	-0.1408	-0.0005	-0.0209	0.0020	0.0068	-0.0363	-0.0095	-0.0040	0.0056	0.0012	-0.0086
mse	0.0027	0.0170	0.0015	0.0015	0.0243	0.0016	0.0070	0.0017	0.0017	0.0105	0.0012	0.0062	0.0012	0.0013	0.0080
var	0.0020	0.0033	0.0014	0.0015	0.0044	0.0016	0.0065	0.0017	0.0017	0.0092	0.0011	0.0062	0.0012	0.0013	0.0079

Table 1.20: Parametric estimation for the SARFIMA$(p,d,q) \times (P,D,Q)_s$ process, when $p = P = 1 = Q$, $q = 0$, $d = 0.2 = D$, $\phi_1 = -0.5 = \Phi_1$, $\Theta_1 = 0.5$, $s \in \{4, 6, 12\}$ and $n = 2000$.

	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\Phi_1}$	$\widehat{\Theta_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\Phi_1}$	$\widehat{\Theta_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\Phi_1}$	$\widehat{\Theta_1}$
$s = 4$															
	W					W-MCMC					BR				
mean	0.2159	0.2021	-0.5059	-0.5028	0.4966	0.2017	0.2030	-0.4998	-0.5004	0.4996	0.1966	0.1953	-0.4970	-0.5009	0.4937
bias	0.0159	0.0021	-0.0059	-0.0028	-0.0034	0.0017	0.0030	0.0002	-0.0004	-0.0004	-0.0034	-0.0047	0.0030	-0.0009	-0.0063
mse	0.0009	0.0056	0.0006	0.0006	0.0057	0.0011	0.0052	0.0011	0.0010	0.0068	0.0005	0.0029	0.0006	0.0006	0.0037
var	0.0007	0.0056	0.0006	0.0006	0.0057	0.0011	0.0052	0.0011	0.0010	0.0068	0.0005	0.0029	0.0006	0.0006	0.0036
$s = 6$															
	W					W-MCMC					BR				
mean	0.2157	0.1888	-0.5053	-0.5026	0.4788	0.2006	0.1993	-0.4977	-0.5003	0.4936	0.1952	0.1957	-0.4960	-0.5011	0.4929
bias	0.0157	-0.0112	-0.0053	-0.0026	-0.0212	0.0006	-0.0007	0.0023	-0.0003	-0.0064	-0.0048	-0.0043	0.0040	-0.0011	-0.0071
mse	0.0009	0.0057	0.0007	0.0006	0.0063	0.0010	0.0050	0.0011	0.0011	0.0072	0.0005	0.0030	0.0006	0.0006	0.0039
var	0.0007	0.0056	0.0006	0.0006	0.0059	0.0010	0.0050	0.0011	0.0011	0.0071	0.0005	0.0030	0.0006	0.0006	0.0039
$s = 12$															
	W					W-MCMC					BR				
mean	0.2170	0.1381	-0.5067	-0.5000	0.4226	0.1986	0.1944	-0.4955	-0.4964	0.4857	0.1957	0.1982	-0.4971	-0.4996	0.4940
bias	0.0170	-0.0619	-0.0067	0.0000	-0.0774	-0.0014	-0.0056	0.0045	0.0036	-0.0143	-0.0043	-0.0018	0.0029	0.0004	-0.0060
mse	0.0010	0.0079	0.0006	0.0006	0.0106	0.0009	0.0054	0.0029	0.0015	0.0082	0.0005	0.0031	0.0005	0.0006	0.0039
var	0.0007	0.0041	0.0006	0.0006	0.0046	0.0009	0.0054	0.0029	0.0015	0.0080	0.0004	0.0031	0.0005	0.0006	0.0039

Table 1.21: Parametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ process, when $p=P=1=q$, $Q=0$, $d=0.2=D$, $\phi_1=-0.5$, $\Phi_1=0.5=\theta_1$, $s\in\{4,6,12\}$ and $n=1000$.

	$s=4$														
	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\theta_1}$	$\widehat{\Phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\theta_1}$	$\widehat{\Phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\theta_1}$	$\widehat{\Phi_1}$
	W					W-MCMC					BR				
mean	0.2845	0.1609	-0.5001	0.5539	0.5260	0.2093	0.1935	-0.5067	0.4946	0.4995	0.1708	0.1853	-0.4995	0.4688	0.5133
bias	0.0845	-0.0391	-0.0001	0.0539	0.0260	0.0093	-0.0065	-0.0067	-0.0054	-0.0005	-0.0292	-0.0147	0.0005	-0.0312	0.0133
mse	0.0181	0.0100	0.0013	0.0126	0.0097	0.0095	0.0074	0.0020	0.0140	0.0088	0.0071	0.0061	0.0035	0.0087	0.0079
var	0.0110	0.0085	0.0013	0.0097	0.0090	0.0094	0.0073	0.0020	0.0140	0.0088	0.0063	0.0059	0.0035	0.0078	0.0078
	$s=6$														
	W					W-MCMC					BR				
mean	0.2927	0.1465	-0.4970	0.5625	0.5387	0.2080	0.1941	-0.5046	0.4905	0.4985	0.1748	0.1864	-0.4995	0.4697	0.5091
bias	0.0927	-0.0535	0.0030	0.0625	0.0387	0.0080	-0.0059	-0.0046	-0.0095	-0.0015	-0.0252	-0.0136	0.0005	-0.0303	0.0091
mse	0.0195	0.0108	0.0012	0.0137	0.0102	0.0091	0.0064	0.0019	0.0143	0.0084	0.0064	0.0060	0.0033	0.0097	0.0094
var	0.0109	0.0079	0.0012	0.0098	0.0087	0.0090	0.0064	0.0019	0.0142	0.0084	0.0058	0.0058	0.0033	0.0088	0.0093
	$s=12$														
	W					W-MCMC					BR				
mean	0.2963	0.0966	-0.4982	0.5683	0.5835	0.2188	0.1838	-0.5065	0.5022	0.5022	0.1759	0.2036	-0.4997	0.4721	0.4902
bias	0.0963	-0.1034	0.0018	0.0683	0.0835	0.0188	-0.0162	-0.0065	0.0022	0.0022	-0.0241	0.0036	0.0003	-0.0279	-0.0098
mse	0.0200	0.0186	0.0012	0.0143	0.0158	0.0101	0.0069	0.0021	0.0149	0.0088	0.0064	0.0061	0.0020	0.0082	0.0096
var	0.0107	0.0079	0.0012	0.0097	0.0088	0.0098	0.0066	0.0020	0.0149	0.0088	0.0059	0.0061	0.0020	0.0075	0.0095

Table 1.22: Parametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ process, when $p=P=1=q$, $Q=0$, $d=0.2=D$, $\phi_1=-0.5$, $\Phi_1=0.5=\theta_1$, $s\in\{4,6,12\}$ and $n=2000$.

	$s=4$														
	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\theta_1}$	$\widehat{\Phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\theta_1}$	$\widehat{\Phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\theta_1}$	$\widehat{\Phi_1}$
	W					W-MCMC					BR				
mean	0.2756	0.1655	-0.5004	0.5580	0.5252	0.2144	0.1919	-0.5049	0.5049	0.5055	0.1854	0.1861	-0.5011	0.4814	0.5151
bias	0.0756	-0.0345	-0.0004	0.0580	0.0252	0.0144	-0.0081	-0.0049	0.0049	0.0055	-0.0146	-0.0139	-0.0011	-0.0186	0.0151
mse	0.0130	0.0058	0.0007	0.0100	0.0054	0.0082	0.0054	0.0014	0.0130	0.0067	0.0040	0.0041	0.0007	0.0048	0.0047
var	0.0073	0.0046	0.0007	0.0067	0.0048	0.0080	0.0053	0.0014	0.0130	0.0066	0.0038	0.0039	0.0007	0.0045	0.0045
	$s=6$														
	W					W-MCMC					BR				
mean	0.2702	0.1609	-0.5001	0.5530	0.5294	0.2123	0.1934	-0.5065	0.5004	0.5025	0.1831	0.1907	-0.5026	0.4796	0.5083
bias	0.0702	-0.0391	-0.0001	0.0530	0.0294	0.0123	-0.0066	-0.0065	0.0004	0.0025	-0.0169	-0.0093	-0.0026	-0.0204	0.0083
mse	0.0116	0.0067	0.0006	0.0091	0.0062	0.0079	0.0049	0.0013	0.0126	0.0061	0.0035	0.0038	0.0008	0.0056	0.0059
var	0.0067	0.0052	0.0006	0.0063	0.0054	0.0077	0.0049	0.0013	0.0126	0.0061	0.0032	0.0037	0.0008	0.0052	0.0059
	$s=12$														
	W					W-MCMC					BR				
mean	0.2800	0.1234	-0.5009	0.5631	0.5662	0.2141	0.1894	-0.5050	0.5022	0.5036	0.1869	0.1955	-0.5003	0.4817	0.4995
bias	0.0800	-0.0766	-0.0009	0.0631	0.0662	0.0141	-0.0106	-0.0050	0.0022	0.0036	-0.0131	-0.0045	-0.0003	-0.0183	-0.0005
mse	0.0128	0.0108	0.0006	0.0100	0.0094	0.0070	0.0047	0.0015	0.0113	0.0063	0.0030	0.0034	0.0015	0.0049	0.0068
var	0.0064	0.0049	0.0006	0.0060	0.0051	0.0068	0.0046	0.0015	0.0113	0.0063	0.0028	0.0034	0.0015	0.0046	0.0068

Table 1.23: Parametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ process, when $p=P=1=q$, $Q=0$, $d=0.2=D$, $\phi_1=-0.5=\theta_1$, $\Phi_1=0.5$, $s\in\{4,6,12\}$ and $n=1000$.

	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\theta_1}$	$\widehat{\Phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\theta_1}$	$\widehat{\Phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\theta_1}$	$\widehat{\Phi_1}$
$s=4$															
	W					W-MCMC					BR				
mean	0.2894	0.1820	-0.5068	0.5674	-0.4856	0.2210	0.1973	-0.5045	0.5126	-0.4928	0.1822	0.1986	-0.5007	0.4803	-0.4927
bias	0.0894	-0.0180	-0.0068	0.0674	0.0144	0.0210	-0.0027	-0.0045	0.0126	0.0072	-0.0178	-0.0014	-0.0007	-0.0197	0.0073
mse	0.0193	0.0018	0.0012	0.0144	0.0014	0.0107	0.0015	0.0016	0.0123	0.0016	0.0085	0.0013	0.0021	0.0100	0.0022
var	0.0114	0.0015	0.0012	0.0098	0.0012	0.0103	0.0015	0.0016	0.0122	0.0016	0.0082	0.0013	0.0021	0.0097	0.0022
$s=6$															
	W					W-MCMC					BR				
mean	0.2894	0.1795	-0.5061	0.5695	-0.4857	0.2173	0.1970	-0.5042	0.5087	-0.4935	0.1769	0.1986	-0.5016	0.4746	-0.4968
bias	0.0894	-0.0205	-0.0061	0.0695	0.0143	0.0173	-0.0030	-0.0042	0.0087	0.0065	-0.0231	-0.0014	-0.0016	-0.0254	0.0032
mse	0.0194	0.0019	0.0012	0.0155	0.0014	0.0103	0.0014	0.0016	0.0121	0.0016	0.0071	0.0011	0.0012	0.0090	0.0016
var	0.0115	0.0014	0.0011	0.0107	0.0012	0.0100	0.0014	0.0015	0.0120	0.0016	0.0065	0.0011	0.0012	0.0083	0.0016
$s=12$															
	W					W-MCMC					BR				
mean	0.2710	0.1697	-0.5047	0.5536	-0.4759	0.2110	0.1961	-0.5021	0.5057	-0.4924	0.1747	0.2023	-0.5019	0.4732	-0.5008
bias	0.0710	-0.0303	-0.0047	0.0536	0.0241	0.0110	-0.0039	-0.0021	0.0057	0.0076	-0.0253	0.0023	-0.0019	-0.0268	-0.0008
mse	0.0163	0.0025	0.0012	0.0139	0.0020	0.0086	0.0014	0.0016	0.0104	0.0015	0.0063	0.0010	0.0011	0.0081	0.0011
var	0.0113	0.0016	0.0011	0.0111	0.0014	0.0085	0.0014	0.0016	0.0103	0.0015	0.0057	0.0010	0.0011	0.0074	0.0011

Table 1.24: Parametric estimation for the SARFIMA$(p,d,q)\times(P,D,Q)_s$ process, when $p=P=1=q$, $Q=0$, $d=0.2=D$, $\phi_1=-0.5=\theta_1$, $\Phi_1=0.5$, $s\in\{4,6,12\}$ and $n=2000$.

	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\theta_1}$	$\widehat{\Phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\theta_1}$	$\widehat{\Phi_1}$	$\widehat{d}$	$\widehat{D}$	$\widehat{\phi_1}$	$\widehat{\theta_1}$	$\widehat{\Phi_1}$
$s=4$															
	W					W-MCMC					BR				
mean	0.2777	0.1891	-0.5052	0.5642	-0.4928	0.2160	0.1992	-0.5038	0.5104	-0.4973	0.1882	0.2014	-0.5019	0.4868	-0.4993
bias	0.0777	-0.0109	-0.0052	0.0642	0.0072	0.0160	-0.0008	-0.0038	0.0104	0.0027	-0.0118	0.0014	-0.0019	-0.0132	0.0007
mse	0.0139	0.0009	0.0007	0.0112	0.0007	0.0073	0.0010	0.0011	0.0085	0.0010	0.0044	0.0007	0.0007	0.0053	0.0007
var	0.0079	0.0008	0.0006	0.0071	0.0006	0.0071	0.0010	0.0011	0.0084	0.0010	0.0043	0.0007	0.0007	0.0052	0.0007
$s=6$															
	W					W-MCMC					BR				
mean	0.2766	0.1884	-0.5034	0.5649	-0.4937	0.2129	0.1981	-0.5017	0.5083	-0.4962	0.1862	0.1994	-0.5012	0.4833	-0.4975
bias	0.0766	-0.0116	-0.0034	0.0649	0.0063	0.0129	-0.0019	-0.0017	0.0083	0.0038	-0.0138	-0.0006	-0.0012	-0.0167	0.0025
mse	0.0130	0.0008	0.0006	0.0110	0.0007	0.0070	0.0009	0.0011	0.0086	0.0011	0.0035	0.0006	0.0006	0.0049	0.0008
var	0.0071	0.0007	0.0006	0.0067	0.0006	0.0068	0.0009	0.0011	0.0086	0.0011	0.0033	0.0006	0.0006	0.0047	0.0008
$s=12$															
	W					W-MCMC					BR				
mean	0.2650	0.1853	-0.5026	0.5548	-0.4896	0.2121	0.1975	-0.5035	0.5070	-0.4944	0.1850	0.2000	-0.5005	0.4846	-0.4980
bias	0.0650	-0.0147	-0.0026	0.0548	0.0104	0.0121	-0.0025	-0.0035	0.0070	0.0056	-0.0150	0.0000	-0.0005	-0.0154	0.0020
mse	0.0108	0.0009	0.0006	0.0095	0.0007	0.0061	0.0009	0.0010	0.0079	0.0011	0.0031	0.0005	0.0005	0.0040	0.0006
var	0.0065	0.0007	0.0006	0.0065	0.0006	0.0059	0.0009	0.0010	0.0078	0.0011	0.0029	0.0005	0.0005	0.0038	0.0006