

Addendum to “Mallows Distance in VARFIMA(0, \mathbf{d} , 0) Processes”

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Introduction and Goals

This report is intended to present an addendum to the article by Lopes, Pumi and Zaniol (2011), “Mallows Distance in VARFIMA(0, \mathbf{d} , 0) Processes”, submitted to the *Communications in Statistics - Simulation and Computation* journal. In the article, the authors apply the Mallows distance as a tool to measure and interpret the dependence among components of VARFIMA(0, \mathbf{d} , 0) processes. To provide grounds for comparison, the well known Kendall’s τ dependence measure is applied as well.

The article is based on extensive Monte Carlo simulations which yield a large amount of information. Due to restrictions in the number of pages, in the article this information is presented only in graphical format. In this addendum, we present the complete simulation results from which the graphs in the article are derived. We provide the results not only in tables, but also in graphical format. The extra graphs presented here contain the same information as the ones presented in the article, but from a different perspective: instead of considering the graphs as a function of the parameter \mathbf{d} , we consider them as a function of the correlation or the copula parameter, whichever applies.

It is not our intention to provide a full account of the results in the article here, but solely to provide more material to facilitate its understanding. Therefore, we refer the reader to the original article for information on the mathematical set up, general and specific definitions, simulation method, conclusions and other information pertinent to the data presented here. We hope that the tables and graphs we compile here will help the reader to better understand the dynamics of the Mallows distance in VARFIMA(0, \mathbf{d} , 0) processes.

The tables and figures presented in this report are organized to match the ones on Lopes, Pumi and Zaniol (2011) as follows:

- Tables and Figures 1 to 4 are related respectively to Figures A.1 to A.4 in Lopes, Pumi and Zaniol (2011). They present the estimation results for the Mallows distance ($\hat{\mathcal{D}}_2$) and the estimates’ standard deviation (sd) in the Gaussian case in both, equal and unequal variances cases. See Subsections 4.1, 4.2 and 4.3 in the article for more details.

- Tables and Figures 5 to 7 are related respectively to Figures A.5 to A.7 in Lopes, Pumi and Zaniol (2011). They present the estimation results for the Mallows distance ($\hat{\mathcal{D}}_2$) and the estimates' standard deviation (sd) in the non-Gaussian case and in the case of heavy tailed marginals. See Subsection 4.4 in the article for more details.
- Tables and Figures 8 to 11 are related respectively to Figures A.8 to A.11 in Lopes, Pumi and Zaniol (2011). They present the estimation results for the Kendall's τ ($\hat{\tau}$) and the estimates' standard deviation (sd) in the Gaussian case in both, equal and unequal variances cases. See Subsections 5.1 and 5.2 in the article for more details.
- Tables and Figures 12 to 14 are related respectively to Figures A.12 to A.14 in Lopes, Pumi and Zaniol (2011). They present the estimation results for the Kendall's τ ($\hat{\tau}$) and the estimates' standard deviation (sd) in the non-Gaussian case and in the case of heavy tailed marginals. See Subsections 5.3 in the article for more details.

Reference

- Lopes, S.R.C.; Pumi, G. and Zaniol, K. (2011). “Mallows Distance in VARFIMA(0, \mathbf{d} , 0) Processes”. Submitted to *Communications in Statistics - Simulation and Computation*.

Table 1: Simulation results for the Mallows distance in the Gaussian case, $\sigma^2 = (1, 1)$.

$\epsilon_t \sim \mathcal{N}_\rho(\mathbf{0}, \sigma^2), \sigma^2 = (1, 1)$															
\mathbf{d}		$\rho = 0$		$\rho = 0.5$		$\rho = 0.95$		\mathbf{d}		$\rho = 0$		$\rho = 0.5$		$\rho = 0.95$	
d_1	d_2	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd	d_1	d_2	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd
-0.4	-0.4	0.0509	0.0116	0.0495	0.0105	0.0309	0.0041	0.1	-0.4	0.0995	0.0236	0.0985	0.0225	0.0963	0.0174
	-0.3	0.0604	0.0163	0.0599	0.0153	0.0467	0.0068		-0.3	0.0766	0.0224	0.0750	0.0213	0.0699	0.0187
	-0.2	0.0783	0.0204	0.0784	0.0188	0.0701	0.0080		-0.2	0.0658	0.0214	0.0633	0.0201	0.0539	0.0191
	-0.1	0.0937	0.0216	0.0941	0.0198	0.0882	0.0089		-0.1	0.0644	0.0218	0.0603	0.0191	0.0467	0.0168
	0.1	0.1015	0.0241	0.1022	0.0229	0.0982	0.0181		0.1	0.0752	0.0320	0.0626	0.0210	0.0323	0.0056
	0.2	0.1103	0.0495	0.1101	0.0485	0.1058	0.0500		0.2	0.1133	0.0566	0.0968	0.0420	0.0708	0.0264
	0.3	0.2005	0.1240	0.1992	0.1188	0.1952	0.1229		0.3	0.2373	0.1162	0.2214	0.0976	0.2031	0.0863
	0.4	0.5008	0.2832	0.4966	0.2633	0.4903	0.2750		0.4	0.5468	0.2692	0.5299	0.2380	0.5123	0.2377
-0.3	-0.4	0.0587	0.0162	0.0573	0.0147	0.0451	0.0068	0.2	-0.4	0.1086	0.0502	0.1077	0.0501	0.1050	0.0501
	-0.3	0.0493	0.0110	0.0479	0.0099	0.0299	0.0039		-0.3	0.0998	0.0520	0.0985	0.0518	0.0938	0.0527
	-0.2	0.0558	0.0143	0.0550	0.0134	0.0411	0.0061		-0.2	0.1017	0.0520	0.0998	0.0510	0.0936	0.0511
	-0.1	0.0668	0.0173	0.0664	0.0162	0.0568	0.0074		-0.1	0.1076	0.0514	0.1041	0.0487	0.0960	0.0463
	0.1	0.0781	0.0225	0.0780	0.0213	0.0714	0.0192		0.1	0.1131	0.0576	0.0976	0.0438	0.0716	0.0277
	0.2	0.1011	0.0511	0.1003	0.0496	0.0943	0.0521		0.2	0.1305	0.0774	0.0997	0.0522	0.0420	0.0138
	0.3	0.2123	0.1199	0.2110	0.1141	0.2071	0.1174		0.3	0.2317	0.1329	0.1941	0.0996	0.1477	0.0632
	0.4	0.5197	0.2763	0.5155	0.2562	0.5094	0.2676		0.4	0.5314	0.2826	0.4944	0.2352	0.4565	0.2149
-0.2	-0.4	0.0587	0.0162	0.0573	0.0147	0.0451	0.0068	0.3	-0.4	0.1980	0.1254	0.1979	0.1253	0.1958	0.1260
	-0.3	0.0493	0.0110	0.0479	0.0099	0.0299	0.0039		-0.3	0.2103	0.1213	0.2100	0.1208	0.2077	0.1207
	-0.2	0.0558	0.0143	0.0550	0.0134	0.0411	0.0061		-0.2	0.2238	0.1170	0.2230	0.1158	0.2201	0.1148
	-0.1	0.0668	0.0173	0.0664	0.0162	0.0568	0.0074		-0.1	0.2345	0.1140	0.2322	0.1113	0.2278	0.1086
	0.1	0.0781	0.0225	0.0780	0.0213	0.0714	0.0192		0.1	0.2362	0.1180	0.2222	0.1042	0.2048	0.0898
	0.2	0.1011	0.0511	0.1003	0.0496	0.0943	0.0521		0.2	0.2309	0.1343	0.1958	0.1039	0.1500	0.0671
	0.3	0.2123	0.1199	0.2110	0.1141	0.2071	0.1174		0.3	0.2717	0.1849	0.1985	0.1273	0.0703	0.0367
	0.4	0.5197	0.2763	0.5155	0.2562	0.5094	0.2676		0.4	0.5217	0.3229	0.4310	0.2435	0.3287	0.1609
-0.1	-0.4	0.0911	0.0214	0.0901	0.0196	0.0862	0.0087	0.4	-0.4	0.4966	0.2823	0.4970	0.2820	0.4961	0.2819
	-0.3	0.0647	0.0173	0.0632	0.0157	0.0550	0.0073		-0.3	0.5157	0.2756	0.5159	0.2751	0.5148	0.2747
	-0.2	0.0513	0.0122	0.0492	0.0108	0.0347	0.0051		-0.2	0.5318	0.2703	0.5316	0.2692	0.5299	0.2683
	-0.1	0.0492	0.0104	0.0467	0.0091	0.0288	0.0038		-0.1	0.5433	0.2668	0.5419	0.2644	0.5390	0.2623
	0.1	0.0645	0.0217	0.0610	0.0182	0.0472	0.0168		0.1	0.5439	0.2694	0.5319	0.2570	0.5183	0.2449
	0.2	0.1078	0.0504	0.1042	0.0461	0.0958	0.0453		0.2	0.5295	0.2826	0.4980	0.2533	0.4636	0.2226
	0.3	0.2359	0.1125	0.2324	0.1045	0.2269	0.1053		0.3	0.5201	0.3241	0.4379	0.2557	0.3375	0.1712
	0.4	0.5470	0.2670	0.5411	0.2455	0.5338	0.2552		0.4	0.6357	0.4473	0.4558	0.3126	0.1478	0.0950

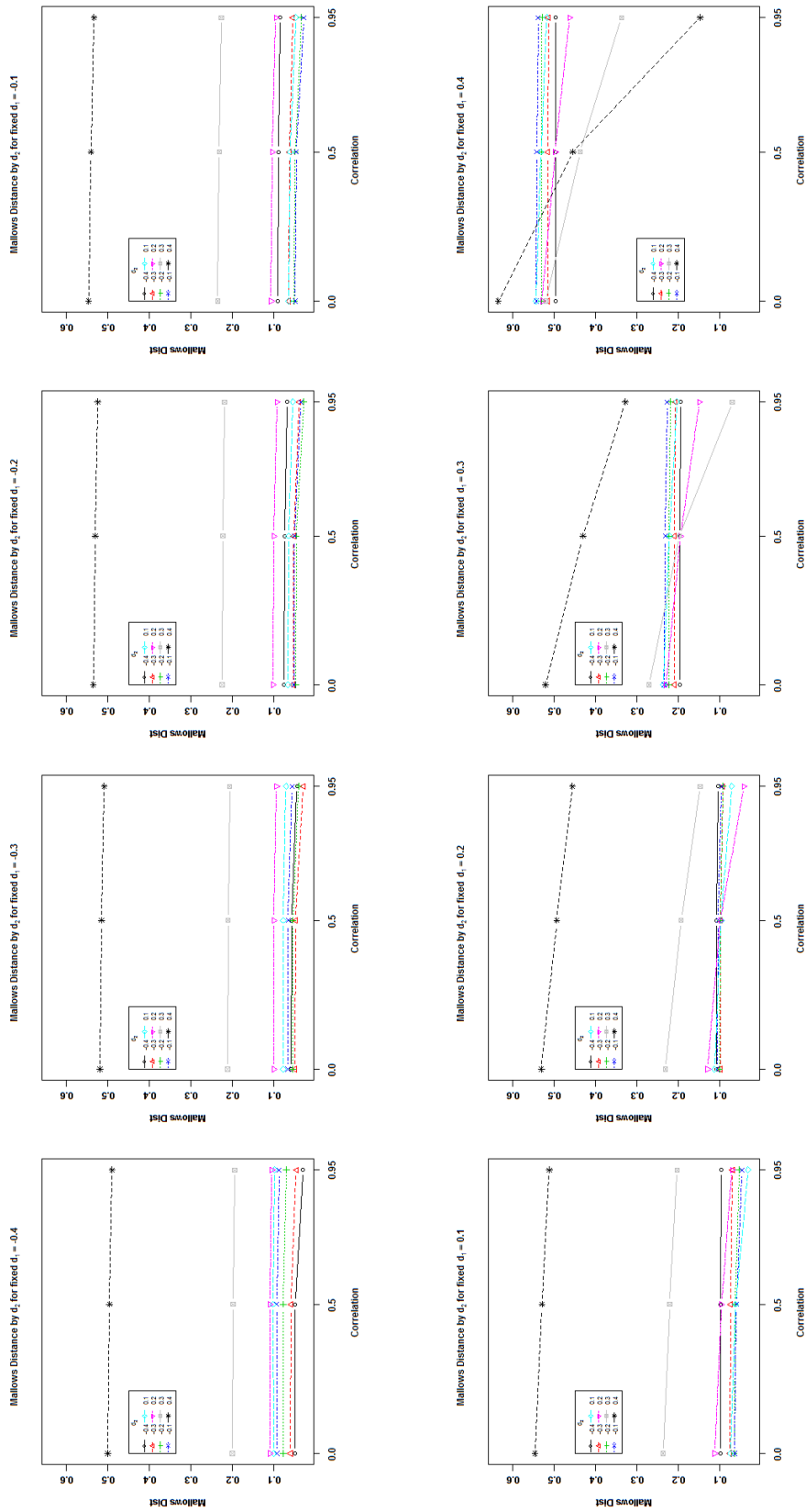


Figure 1: Plots of Mallows distance by correlation for fixed d_1 . Gaussian case with equal variances.

Table 2: Simulation results for the Mallows distance in the Gaussian case, $\sigma^2 = (1, 2)$.

$\epsilon_t \sim \mathcal{N}_\rho(\mathbf{0}, \sigma^2), \sigma^2 = (1, 2)$																																																																																																																							
d	$\rho = 0$			$\rho = 0.5$			$\rho = 0.95$			d	$\rho = 0$			$\rho = 0.5$			$\rho = 0.95$																																																																																																						
-0.4	-0.4	0.4529	0.0318	0.4515	0.0290	0.4519	0.0126	0.1	-0.4	0.5317	0.0302	0.5303	0.0285	0.5317	0.0180	-0.3	0.4048	0.0306	0.4033	0.0279	0.4036	0.0116	-0.3	0.4836	0.0290	0.4821	0.0272	0.4832	0.0163	-0.2	0.3668	0.0296	0.3653	0.0270	0.3655	0.0111	-0.2	0.4458	0.0280	0.4439	0.0260	0.4445	0.0145	-0.1	0.3415	0.0289	0.3398	0.0263	0.3400	0.0113	-0.1	0.4203	0.0273	0.4179	0.0252	0.4177	0.0128	0.1	0.3518	0.0301	0.3502	0.0280	0.3505	0.0172	0.1	0.4294	0.0293	0.4242	0.0258	0.4207	0.0111	0.2	0.4206	0.0443	0.4194	0.0421	0.4193	0.0373	0.2	0.4955	0.0429	0.4881	0.0355	0.4816	0.0211	0.3	0.5956	0.1100	0.5948	0.1036	0.5933	0.1060	0.3	0.6653	0.1042	0.6556	0.0889	0.6458	0.0796	0.4	1.0167	0.3156	1.0129	0.2905	1.0077	0.3055	0.4	1.0790	0.3046	1.0651	0.2701	1.0510	0.2735						
	-0.3	-0.4	0.4869	0.0312	0.4856	0.0286	0.4861		0.0131	0.2	-0.4	0.5037	0.0338	0.5023	0.0327	0.5038	0.0259	-0.3	0.4387	0.0300	0.4373	0.0274	0.4376	0.0119	-0.3	0.4565	0.0333	0.4549	0.0320	0.4560	0.0251	-0.2	0.4007	0.0290	0.3992	0.0265	0.3994	0.0111	-0.2	0.4194	0.0330	0.4172	0.0313	0.4177	0.0239	-0.1	0.3753	0.0283	0.3735	0.0258	0.3737	0.0109	-0.1	0.3946	0.0329	0.3913	0.0305	0.3906	0.0218	0.1	0.3851	0.0294	0.3833	0.0273	0.3835	0.0161	0.1	0.4038	0.0369	0.3944	0.0295	0.3867	0.0131	0.2	0.4528	0.0428	0.4514	0.0404	0.4512	0.0351	0.2	0.4693	0.0525	0.4539	0.0376	0.4390	0.0141	0.3	0.6257	0.1065	0.6247	0.1001	0.6231	0.1020	0.3	0.6390	0.1144	0.6169	0.0867	0.5943	0.0603	0.4	1.0438	0.3101	1.0399	0.2852	1.0348	0.2998	0.4	1.0540	0.3147	1.0244	0.2653	0.9956	0.2509				
		-0.2	-0.4	0.5139	0.0307	0.5125	0.0283		0.5133		0.0137	0.3	-0.4	0.4639	0.0760	0.4626	0.0758	0.4639	0.0740	-0.3	0.4657	0.0295	0.4642	0.0271	0.4647	0.0123	-0.3	0.4209	0.0800	0.4193	0.0795	0.4201	0.0776	-0.2	0.4276	0.0285	0.4261	0.0261	0.4263	0.0113	-0.2	0.3878	0.0837	0.3853	0.0825	0.3852	0.0800	-0.1	0.4021	0.0278	0.4003	0.0254	0.4005	0.0108	-0.1	0.3661	0.0866	0.3618	0.0837	0.3598	0.0795	0.1	0.4115	0.0289	0.4096	0.0267	0.4094	0.0150	0.1	0.3750	0.0909	0.3584	0.0751	0.3429	0.0564	0.2	0.4785	0.0416	0.4768	0.0391	0.4762	0.0330	0.2	0.4355	0.1019	0.4036	0.0693	0.3692	0.0275	0.3	0.6498	0.1038	0.6484	0.0971	0.6465	0.0983	0.3	0.5982	0.1552	0.5460	0.0999	0.4869	0.0288	0.4	1.0654	0.3058	1.0612	0.2808	1.0559	0.2948	0.4	1.0089	0.3475	0.9391	0.2635	0.8680	0.1970		
			-0.1	-0.4	0.5323	0.0304	0.5310		0.0281		0.5320		0.0146	0.4	-0.4	0.5154	0.2727	0.5145	0.2729	0.5153	0.2724	-0.3	0.4841	0.0292	0.4826	0.0269	0.4833	0.0130	-0.3	0.4895	0.2817	0.4883	0.2817	0.4885	0.2810	-0.2	0.4460	0.0282	0.4444	0.0258	0.4448	0.0117	-0.2	0.4717	0.2886	0.4697	0.2878	0.4687	0.2866	-0.1	0.4204	0.0274	0.4186	0.0251	0.4187	0.0108	-0.1	0.4614	0.2931	0.4574	0.2906	0.4541	0.2879	0.1	0.4296	0.0285	0.4272	0.0262	0.4266	0.0138	0.1	0.4684	0.2953	0.4474	0.2785	0.4258	0.2623	0.2	0.4961	0.0410	0.4937	0.0378	0.4923	0.0305	0.2	0.5058	0.2969	0.4575	0.2564	0.4036	0.2129	0.3	0.6663	0.1022	0.6640	0.0945	0.6612	0.0945	0.3	0.6266	0.3209	0.5249	0.2297	0.3942	0.1068	0.4	1.0802	0.3030	1.0751	0.2771	1.0689	0.2899	0.4	0.9897	0.4654	0.8217	0.3109	0.6115	0.0887

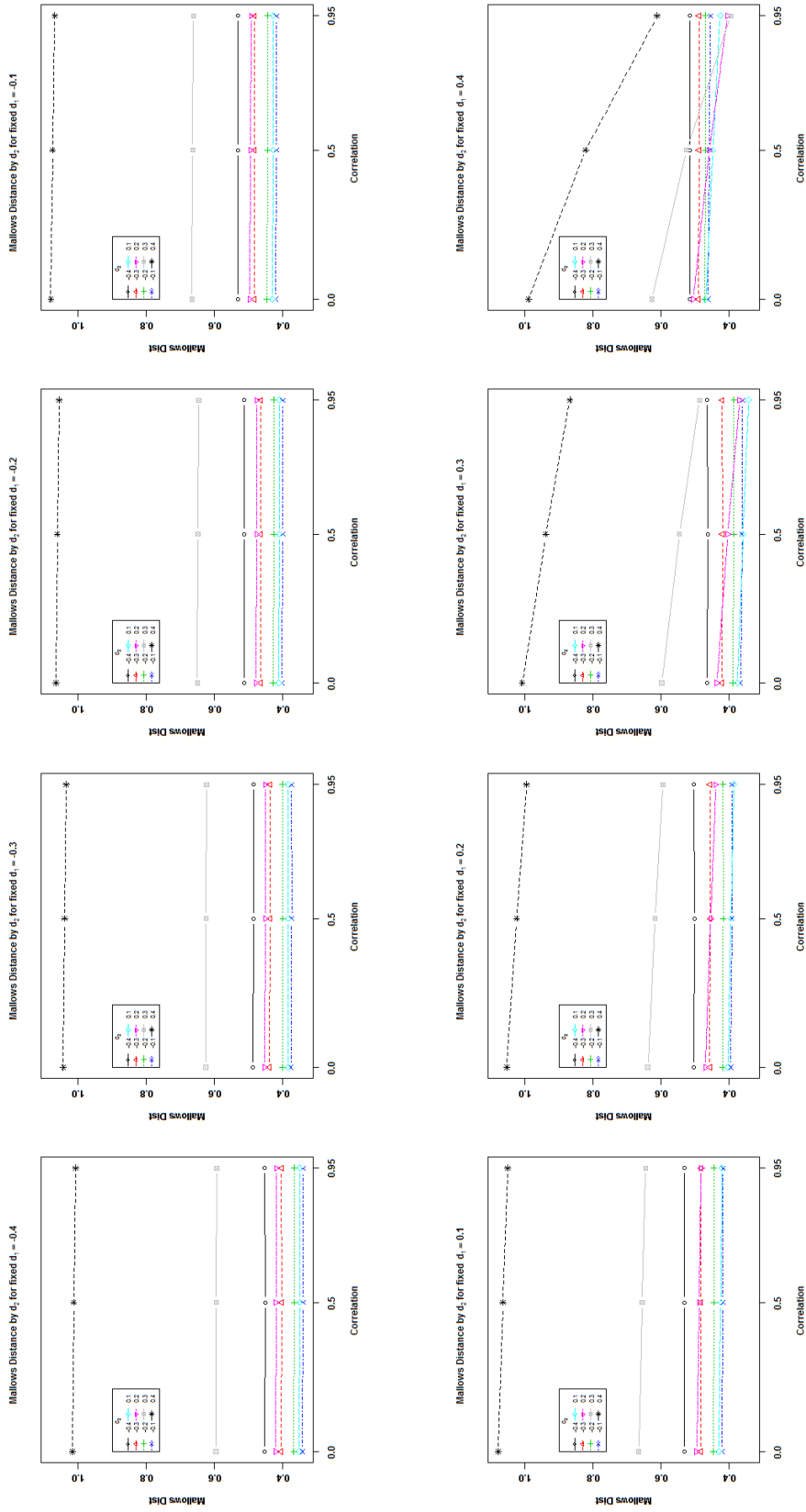
Figure 2: Plots of Mallows distance by correlation for fixed d_1 for $\sigma^2 = (1, 2)$.

Table 3: Simulation results for the Mallows distance in the Gaussian case, $\sigma^2 = (2, 3)$.

$\epsilon_t \sim \mathcal{N}_\rho(\mathbf{0}, \sigma^2), \sigma^2 = (2, 3)$															
\mathbf{d}		$\rho = 0$		$\rho = 0.5$		$\rho = 0.95$		\mathbf{d}		$\rho = 0$		$\rho = 0.5$		$\rho = 0.95$	
d_1	d_2	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd	d_1	d_2	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd
-0.4	-0.4	0.3513	0.0407	0.3496	0.0369	0.3482	0.0144	0.1	-0.4	0.4634	0.0386	0.4617	0.0361	0.4628	0.0222
	-0.3	0.2933	0.0391	0.2914	0.0354	0.2894	0.0136		-0.3	0.4054	0.0372	0.4034	0.0346	0.4038	0.0203
	-0.2	0.2480	0.0376	0.2460	0.0340	0.2436	0.0133		-0.2	0.3598	0.0361	0.3572	0.0333	0.3566	0.0183
	-0.1	0.2184	0.0363	0.2161	0.0329	0.2136	0.0138		-0.1	0.3296	0.0354	0.3257	0.0322	0.3234	0.0159
	0.1	0.2368	0.0395	0.2349	0.0367	0.2335	0.0254		0.1	0.3434	0.0402	0.3338	0.0335	0.3247	0.0128
	0.2	0.3291	0.0667	0.3277	0.0636	0.3264	0.0601		0.2	0.4294	0.0631	0.4160	0.0493	0.4025	0.0277
	0.3	0.5564	0.1604	0.5552	0.1513	0.5523	0.1553		0.3	0.6482	0.1488	0.6319	0.1251	0.6149	0.1108
	0.4	1.0890	0.4222	1.0834	0.3897	1.0757	0.4089		0.4	1.1712	0.4045	1.1500	0.3578	1.1287	0.3602
-0.3	-0.4	0.3987	0.0399	0.3971	0.0363	0.3963	0.0148	0.2	-0.4	0.4314	0.0493	0.4297	0.0479	0.4309	0.0409
	-0.3	0.3403	0.0384	0.3385	0.0349	0.3372	0.0136		-0.3	0.3762	0.0504	0.3740	0.0487	0.3744	0.0420
	-0.2	0.2946	0.0371	0.2926	0.0336	0.2907	0.0130		-0.2	0.3335	0.0519	0.3303	0.0495	0.3293	0.0423
	-0.1	0.2643	0.0360	0.2620	0.0326	0.2600	0.0131		-0.1	0.3056	0.0534	0.3002	0.0493	0.2965	0.0402
	0.1	0.2807	0.0385	0.2785	0.0356	0.2771	0.0226		0.1	0.3190	0.0613	0.3012	0.0455	0.2827	0.0199
	0.2	0.3709	0.0628	0.3692	0.0594	0.3677	0.0547		0.2	0.4015	0.0845	0.3727	0.0562	0.3409	0.0164
	0.3	0.5953	0.1535	0.5938	0.1443	0.5908	0.1477		0.3	0.6172	0.1674	0.5793	0.1238	0.5392	0.0809
	0.4	1.1244	0.4133	1.1187	0.3810	1.1112	0.3997		0.4	1.1397	0.4207	1.0942	0.3521	1.0495	0.3278
-0.2	-0.4	0.4365	0.0393	0.4349	0.0359	0.4347	0.0156	0.3	-0.4	0.4113	0.1405	0.4099	0.1402	0.4106	0.1387
	-0.3	0.3779	0.0378	0.3761	0.0344	0.3753	0.0140		-0.3	0.3686	0.1497	0.3666	0.1491	0.3663	0.1475
	-0.2	0.3319	0.0365	0.3299	0.0332	0.3284	0.0129		-0.2	0.3380	0.1576	0.3347	0.1560	0.3326	0.1538
	-0.1	0.3013	0.0355	0.2990	0.0322	0.2971	0.0126		-0.1	0.3198	0.1632	0.3134	0.1592	0.3077	0.1549
	0.1	0.3164	0.0378	0.3137	0.0346	0.3119	0.0202		0.1	0.3309	0.1681	0.3035	0.1452	0.2732	0.1208
	0.2	0.4049	0.0601	0.4026	0.0561	0.4005	0.0499		0.2	0.3948	0.1784	0.3402	0.1291	0.2730	0.0649
	0.3	0.6269	0.1484	0.6247	0.1387	0.6212	0.1410		0.3	0.5852	0.2392	0.4974	0.1553	0.3861	0.0358
	0.4	1.1529	0.4065	1.1467	0.3738	1.1389	0.3917		0.4	1.0899	0.4729	0.9809	0.3550	0.8669	0.2506
-0.1	-0.4	0.4624	0.0387	0.4608	0.0356	0.4612	0.0167	0.4	-0.4	0.6169	0.4312	0.6165	0.4312	0.6158	0.4313
	-0.3	0.4038	0.0373	0.4020	0.0340	0.4017	0.0149		-0.3	0.6132	0.4337	0.6125	0.4333	0.6110	0.4333
	-0.2	0.3576	0.0360	0.3555	0.0328	0.3545	0.0134		-0.2	0.6164	0.4329	0.6148	0.4316	0.6120	0.4309
	-0.1	0.3269	0.0350	0.3243	0.0318	0.3225	0.0124		-0.1	0.6219	0.4311	0.6179	0.4276	0.6123	0.4250
	0.1	0.3412	0.0373	0.3376	0.0338	0.3348	0.0176		0.1	0.6267	0.4352	0.6016	0.4140	0.5730	0.3952
	0.2	0.4286	0.0585	0.4248	0.0534	0.4214	0.0449		0.2	0.6373	0.4494	0.5743	0.3996	0.5009	0.3499
	0.3	0.6487	0.1451	0.6449	0.1339	0.6400	0.1343		0.3	0.7270	0.4891	0.5811	0.3743	0.3831	0.2274
	0.4	1.1725	0.4020	1.1649	0.3680	1.1558	0.3842		0.4	1.1190	0.6557	0.8665	0.4450	0.5122	0.1110

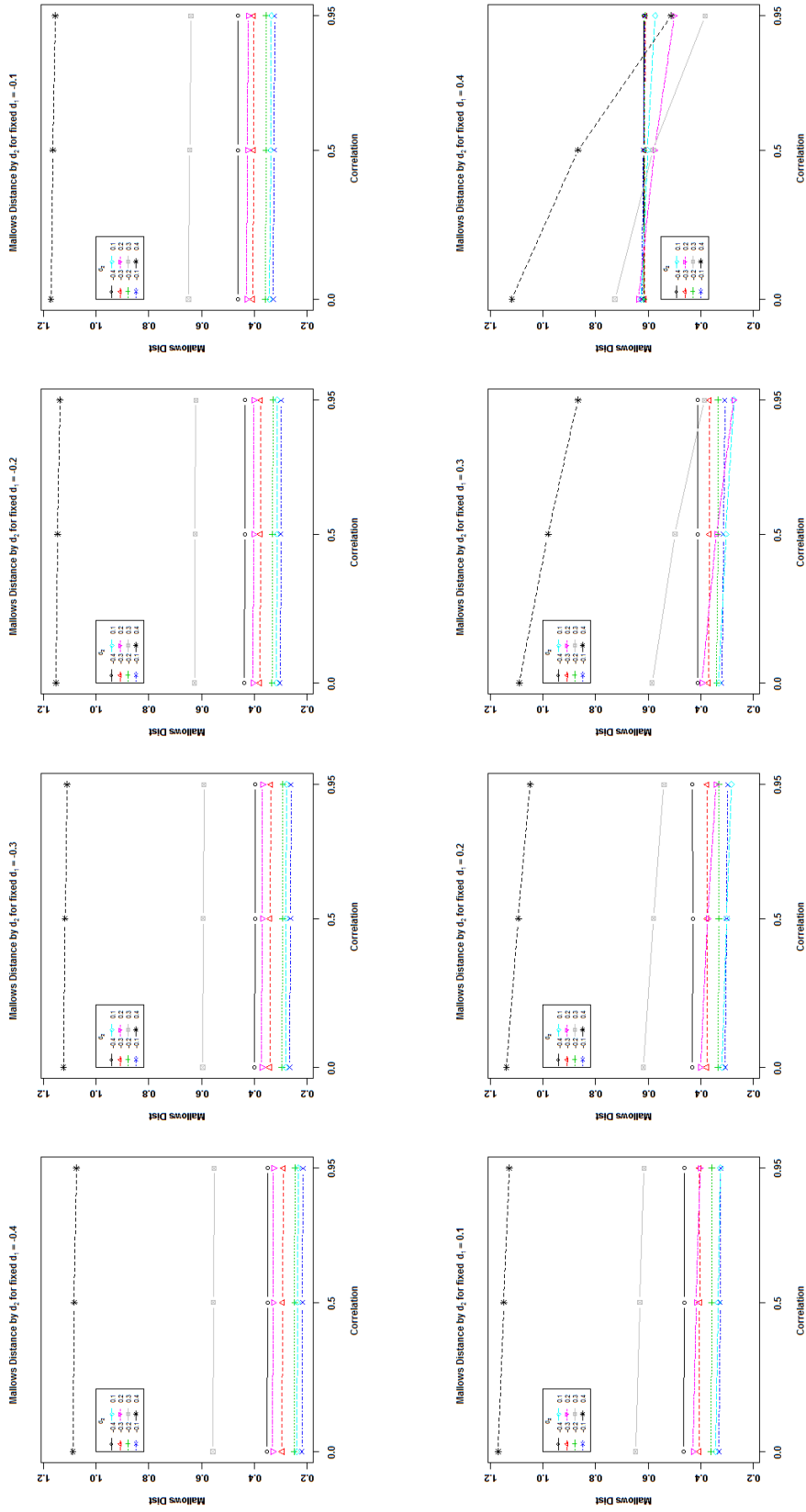


Figure 3: Plots of Mallows distance by correlation for fixed d_1 for the case $\sigma^2 = (2, 3)$.

Table 4: Simulation results for the Mallows distance in the Gaussian case, $\sigma^2 = (1, 3)$.

$\epsilon_t \sim \mathcal{N}_\rho(\mathbf{0}, \sigma^2), \sigma^2 = (1, 3)$															
\mathbf{d}		$\rho = 0$		$\rho = 0.5$		$\rho = 0.95$		\mathbf{d}		$\rho = 0$		$\rho = 0.5$		$\rho = 0.95$	
d_1	d_2	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd	d_1	d_2	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd
-0.4	-0.4	0.7982	0.0367	0.7966	0.0339	0.7979	0.0174	0.1	-0.4	0.8765	0.0352	0.8748	0.0336	0.8769	0.0228
	-0.3	0.7390	0.0352	0.7373	0.0325	0.7386	0.0159		-0.3	0.8174	0.0337	0.8156	0.0320	0.8174	0.0206
	-0.2	0.6923	0.0340	0.6905	0.0313	0.6917	0.0149		-0.2	0.7706	0.0324	0.7686	0.0305	0.7700	0.0186
	-0.1	0.6609	0.0330	0.6589	0.0304	0.6601	0.0147		-0.1	0.7392	0.0315	0.7366	0.0294	0.7375	0.0167
	0.1	0.6709	0.0341	0.6689	0.0320	0.6700	0.0198		0.1	0.7487	0.0330	0.7441	0.0301	0.7424	0.0154
	0.2	0.7493	0.0466	0.7478	0.0445	0.7482	0.0378		0.2	0.8255	0.0455	0.8194	0.0399	0.8151	0.0259
	0.3	0.9503	0.1137	0.9497	0.1073	0.9484	0.1090		0.3	1.0229	0.1098	1.0149	0.0959	1.0068	0.0873
	0.4	1.4445	0.3502	1.4408	0.3219	1.4356	0.3391		0.4	1.5101	0.3409	1.4974	0.3038	1.4841	0.3098
-0.3	-0.4	0.8324	0.0362	0.8307	0.0336	0.8322	0.0180	0.2	-0.4	0.8452	0.0371	0.8435	0.0360	0.8457	0.0279
	-0.3	0.7732	0.0347	0.7715	0.0321	0.7728	0.0164		-0.3	0.7864	0.0358	0.7845	0.0345	0.7864	0.0260
	-0.2	0.7264	0.0334	0.7246	0.0309	0.7258	0.0152		-0.2	0.7400	0.0347	0.7377	0.0332	0.7391	0.0240
	-0.1	0.6950	0.0325	0.6929	0.0300	0.6941	0.0147		-0.1	0.7088	0.0340	0.7057	0.0320	0.7062	0.0217
	0.1	0.7048	0.0335	0.7027	0.0314	0.7036	0.0192		0.1	0.7184	0.0367	0.7109	0.0318	0.7065	0.0160
	0.2	0.7825	0.0456	0.7810	0.0435	0.7812	0.0365		0.2	0.7954	0.0512	0.7833	0.0405	0.7731	0.0194
	0.3	0.9820	0.1113	0.9812	0.1049	0.9799	0.1062		0.3	0.9934	0.1172	0.9753	0.0928	0.9572	0.0700
	0.4	1.4732	0.3455	1.4695	0.3174	1.4643	0.3342		0.4	1.4827	0.3494	1.4559	0.2983	1.4296	0.2877
-0.2	-0.4	0.8594	0.0357	0.8578	0.0334	0.8594	0.0188	0.3	-0.4	0.7881	0.0600	0.7864	0.0597	0.7885	0.0567
	-0.3	0.8002	0.0342	0.7985	0.0318	0.7999	0.0170		-0.3	0.7312	0.0612	0.7293	0.0607	0.7310	0.0575
	-0.2	0.7535	0.0329	0.7516	0.0305	0.7529	0.0156		-0.2	0.6866	0.0624	0.6840	0.0613	0.6850	0.0576
	-0.1	0.7220	0.0320	0.7199	0.0296	0.7210	0.0148		-0.1	0.6568	0.0634	0.6526	0.0610	0.6523	0.0557
	0.1	0.7316	0.0330	0.7294	0.0309	0.7301	0.0185		0.1	0.6666	0.0678	0.6531	0.0551	0.6426	0.0373
	0.2	0.8089	0.0449	0.8071	0.0427	0.8071	0.0351		0.2	0.7422	0.0826	0.7169	0.0566	0.6925	0.0221
	0.3	1.0072	0.1094	1.0061	0.1028	1.0044	0.1035		0.3	0.9386	0.1471	0.8959	0.0984	0.8511	0.0394
	0.4	1.4961	0.3419	1.4921	0.3137	1.4867	0.3299		0.4	1.4288	0.3772	1.3664	0.2926	1.3042	0.2352
-0.1	-0.4	0.8779	0.0354	0.8763	0.0333	0.8780	0.0198	0.4	-0.4	0.7623	0.2138	0.7609	0.2141	0.7626	0.2133
	-0.3	0.8187	0.0339	0.8170	0.0317	0.8185	0.0178		-0.3	0.7150	0.2224	0.7132	0.2224	0.7145	0.2213
	-0.2	0.7719	0.0326	0.7700	0.0303	0.7713	0.0161		-0.2	0.6788	0.2296	0.6761	0.2288	0.6764	0.2270
	-0.1	0.7404	0.0316	0.7383	0.0293	0.7393	0.0150		-0.1	0.6552	0.2348	0.6503	0.2320	0.6485	0.2284
	0.1	0.7499	0.0327	0.7474	0.0305	0.7477	0.0176		0.1	0.6648	0.2377	0.6435	0.2186	0.6242	0.1987
	0.2	0.8269	0.0445	0.8246	0.0418	0.8239	0.0333		0.2	0.7293	0.2414	0.6829	0.1961	0.6344	0.1456
	0.3	1.0244	0.1083	1.0225	0.1009	1.0201	0.1004		0.3	0.9051	0.2808	0.8118	0.1856	0.7012	0.0599
	0.4	1.5117	0.3395	1.5068	0.3105	1.5006	0.3255		0.4	1.3719	0.4776	1.2194	0.3199	1.0464	0.1225

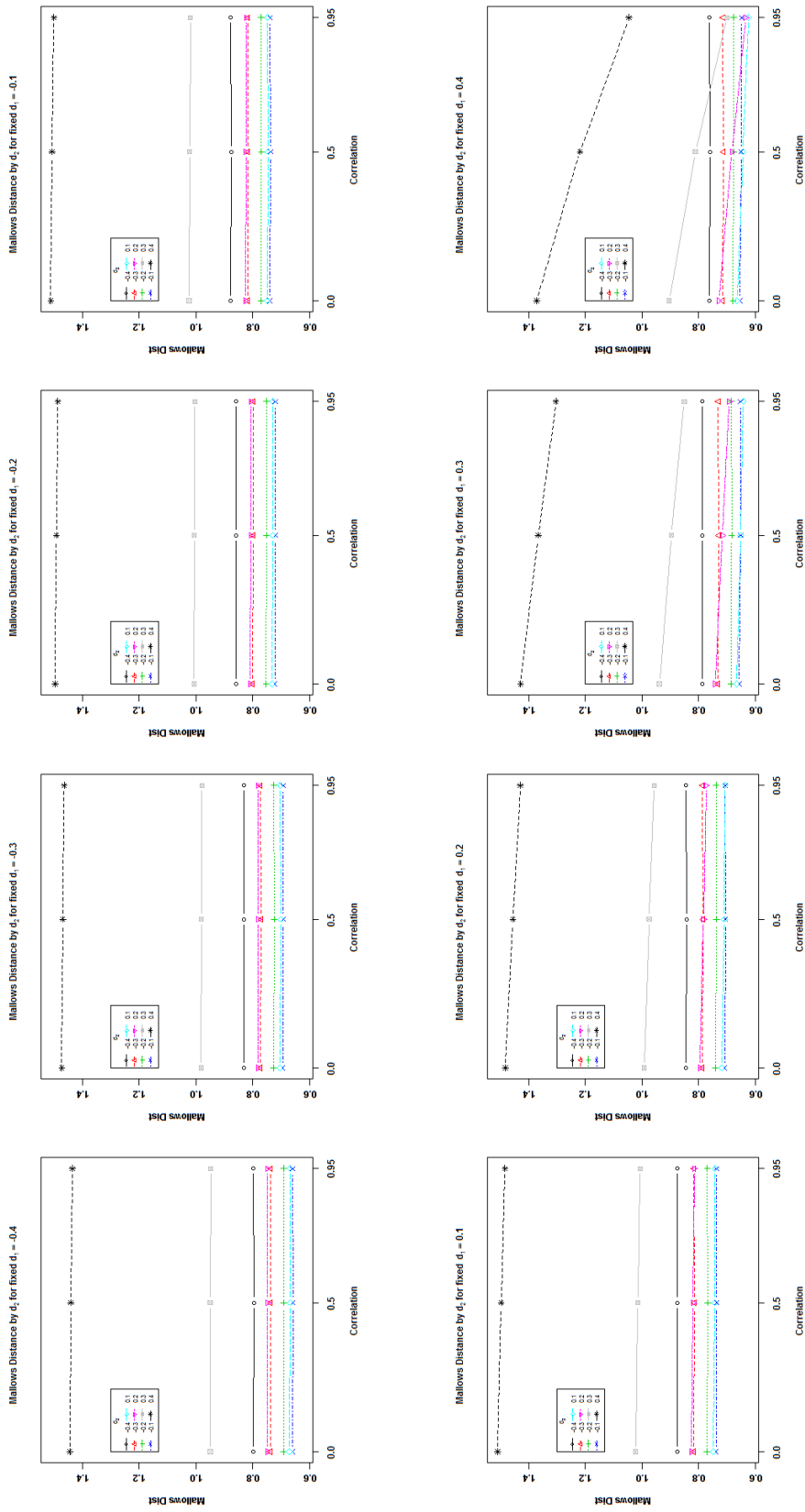


Figure 4: Plots of Mallows distance by correlation for fixed d_1 for the case $\sigma^2 = (1, 3)$.

Table 5: Simulation results for the Mallows distance in the Frank- $\mathcal{N}(0, 1)$ case.

$\varepsilon_t \sim \text{Frank-}\mathcal{N}(0, 1)$															
\mathbf{d}		$\theta = 0$		$\theta = 3.3$		$\theta = 18$		\mathbf{d}		$\theta = 0$		$\theta = 3.3$		$\theta = 18$	
d_1	d_2	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd	d_1	d_2	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd
-0.4	-0.4	0.0510	0.0115	0.0502	0.0113	0.0430	0.0085	0.1	-0.4	0.0940	0.0224	0.1103	0.0220	0.1061	0.0196
	-0.3	0.0655	0.0168	0.0531	0.0127	0.0501	0.0100		-0.3	0.0720	0.0212	0.0834	0.0213	0.0793	0.0196
	-0.2	0.0858	0.0191	0.0689	0.0168	0.0690	0.0124		-0.2	0.0631	0.0202	0.0674	0.0205	0.0628	0.0189
	-0.1	0.1022	0.0196	0.0840	0.0183	0.0853	0.0135		-0.1	0.0633	0.0203	0.0608	0.0193	0.0546	0.0165
	0.1	0.1100	0.0222	0.0939	0.0223	0.0960	0.0206		0.1	0.0724	0.0298	0.0631	0.0207	0.0452	0.0099
	0.2	0.1154	0.0493	0.1111	0.0539	0.1101	0.0541		0.2	0.1069	0.0557	0.1068	0.0430	0.0860	0.0311
	0.3	0.1960	0.1248	0.2232	0.1310	0.2091	0.1299		0.3	0.2278	0.1147	0.2468	0.1063	0.2193	0.0936
	0.4	0.4823	0.2782	0.5440	0.2957	0.5058	0.2799		0.4	0.5271	0.2624	0.5754	0.2682	0.5268	0.2423
-0.3	-0.4	0.0542	0.0131	0.0653	0.0164	0.0586	0.0122	0.2	-0.4	0.1093	0.0514	0.1155	0.0489	0.1129	0.0487
	-0.3	0.0490	0.0108	0.0483	0.0107	0.0420	0.0083		-0.3	0.1034	0.0523	0.1015	0.0515	0.0994	0.0511
	-0.2	0.0595	0.0148	0.0493	0.0112	0.0460	0.0090		-0.2	0.1075	0.0509	0.0986	0.0514	0.0966	0.0501
	-0.1	0.0725	0.0167	0.0582	0.0140	0.0574	0.0109		-0.1	0.1142	0.0490	0.1006	0.0492	0.0973	0.0461
	0.1	0.0836	0.0218	0.0721	0.0214	0.0724	0.0209		0.1	0.1159	0.0529	0.0930	0.0437	0.0745	0.0283
	0.2	0.1018	0.0521	0.1057	0.0544	0.1021	0.0555		0.2	0.1262	0.0727	0.1029	0.0521	0.0594	0.0198
	0.3	0.2056	0.1210	0.2365	0.1256	0.2217	0.1244		0.3	0.2194	0.1292	0.2177	0.1054	0.1674	0.0729
	0.4	0.5007	0.2712	0.5628	0.2885	0.5245	0.2725		0.4	0.5094	0.2735	0.5395	0.2628	0.4723	0.2209
-0.2	-0.4	0.0698	0.0174	0.0860	0.0186	0.0803	0.0138	0.3	-0.4	0.2203	0.1295	0.1957	0.1247	0.1963	0.1242
	-0.3	0.0502	0.0116	0.0593	0.0145	0.0531	0.0113		-0.3	0.2336	0.1242	0.2050	0.1208	0.2059	0.1198
	-0.2	0.0476	0.0103	0.0469	0.0100	0.0411	0.0082		-0.2	0.2471	0.1191	0.2165	0.1158	0.2169	0.1141
	-0.1	0.0544	0.0126	0.0467	0.0097	0.0427	0.0082		-0.1	0.2572	0.1153	0.2250	0.1112	0.2239	0.1081
	0.1	0.0685	0.0216	0.0629	0.0202	0.0605	0.0204		0.1	0.2544	0.1157	0.2139	0.1034	0.2004	0.0895
	0.2	0.0999	0.0526	0.1097	0.0523	0.1041	0.0533		0.2	0.2422	0.1285	0.1883	0.1038	0.1487	0.0693
	0.3	0.2178	0.1166	0.2497	0.1202	0.2342	0.1184		0.3	0.2685	0.1753	0.2113	0.1301	0.1018	0.0517
	0.4	0.5165	0.2655	0.5782	0.2825	0.5396	0.2660		0.4	0.4970	0.3123	0.4779	0.2647	0.3533	0.1751
-0.1	-0.4	0.0849	0.0190	0.1024	0.0191	0.0973	0.0144	0.4	-0.4	0.5506	0.3005	0.4819	0.2784	0.4839	0.2777
	-0.3	0.0592	0.0146	0.0725	0.0164	0.0671	0.0126		-0.3	0.5693	0.2933	0.5001	0.2712	0.5018	0.2703
	-0.2	0.0478	0.0102	0.0538	0.0125	0.0479	0.0101		-0.2	0.5848	0.2874	0.5154	0.2650	0.5165	0.2637
	-0.1	0.0481	0.0100	0.0468	0.0097	0.0407	0.0082		-0.1	0.5954	0.2831	0.5255	0.2601	0.5252	0.2576
	0.1	0.0635	0.0218	0.0621	0.0191	0.0567	0.0181		0.1	0.5907	0.2821	0.5147	0.2522	0.5038	0.2404
	0.2	0.1038	0.0517	0.1152	0.0489	0.1076	0.0487		0.2	0.5698	0.2908	0.4801	0.2491	0.4499	0.2199
	0.3	0.2280	0.1132	0.2587	0.1153	0.2420	0.1122		0.3	0.5482	0.3222	0.4259	0.2557	0.3326	0.1771
	0.4	0.5279	0.2617	0.5880	0.2774	0.5485	0.2599		0.4	0.6362	0.4309	0.4909	0.3252	0.2145	0.1331

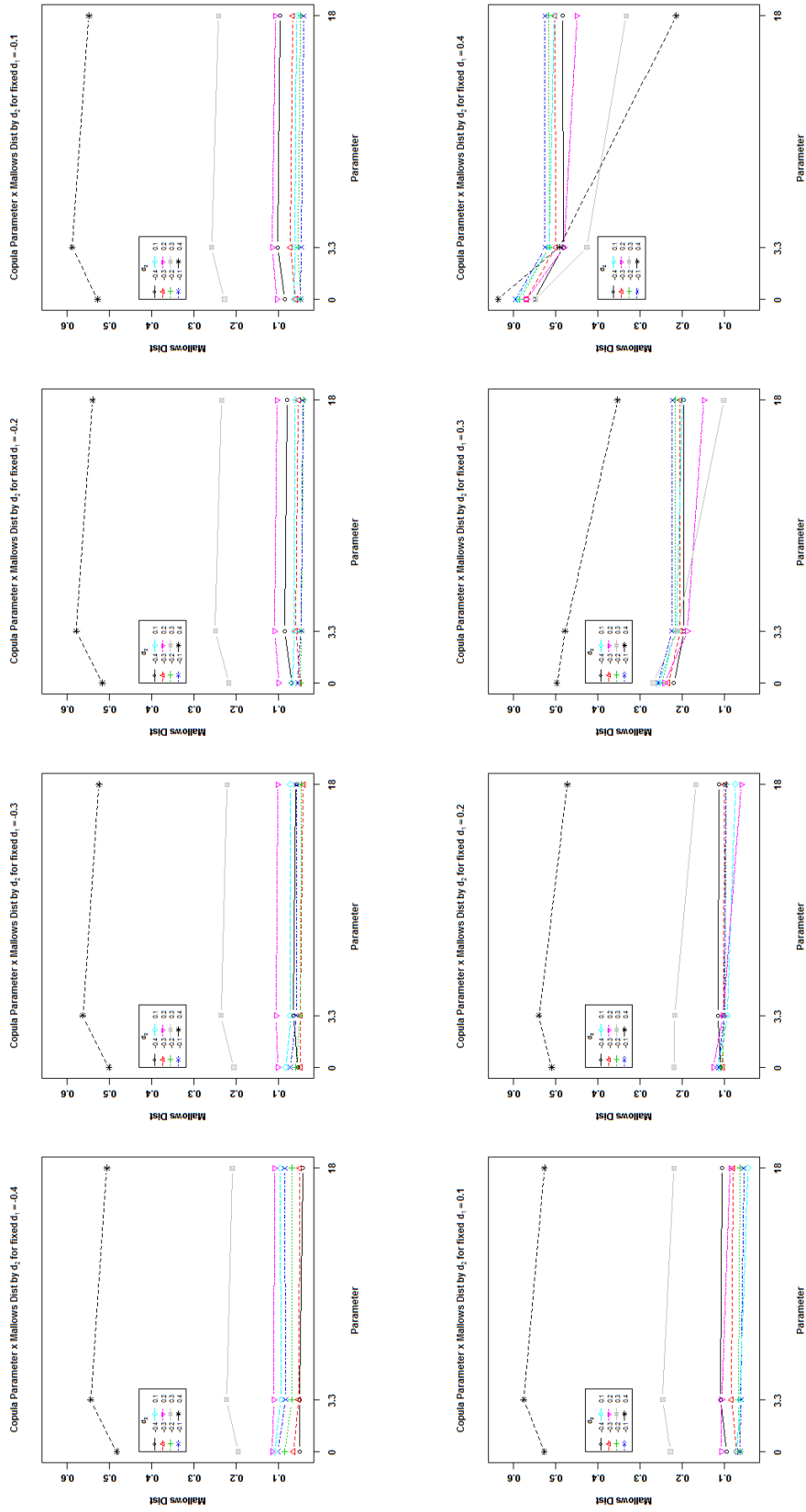


Figure 5: Plots of Mallows distance by the Frank copula’s parameter $\theta \in \{0, 3.3, 18\}$ for fixed d_1 . Marginals were taken to be standard normal.

Table 6: Simulation results for the Mallows distance in the Frank- t_3 case

$\varepsilon_t \sim \text{Frank-}t_3$															
\mathbf{d}		$\theta = 0$		$\theta = 3.3$		$\theta = 18$		\mathbf{d}		$\theta = 0$		$\theta = 3.3$		$\theta = 18$	
d_1	d_2	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd	d_1	d_2	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd
-0.4	-0.4	0.2552	0.2165	0.2550	0.2036	0.2567	0.1980	0.1	-0.4	0.2739	0.2056	0.2847	0.1953	0.2870	0.1890
	-0.3	0.2609	0.2141	0.2558	0.2012	0.2573	0.1956		-0.3	0.2641	0.2070	0.2707	0.1967	0.2722	0.1909
	-0.2	0.2707	0.2116	0.2622	0.1983	0.2639	0.1929		-0.2	0.2607	0.2079	0.2633	0.1976	0.2643	0.1925
	-0.1	0.2803	0.2097	0.2697	0.1957	0.2719	0.1906		-0.1	0.2612	0.2081	0.2607	0.1979	0.2613	0.1935
	0.1	0.2835	0.2090	0.2737	0.1949	0.2760	0.1897		0.1	0.2651	0.2078	0.2628	0.1979	0.2603	0.1942
	0.2	0.2847	0.2111	0.2819	0.1965	0.2832	0.1916		0.2	0.2827	0.2070	0.2854	0.1952	0.2795	0.1912
	0.3	0.3433	0.2232	0.3594	0.2101	0.3562	0.2050		0.3	0.3683	0.2141	0.3849	0.2000	0.3734	0.1925
	0.4	0.5977	0.3182	0.6383	0.3209	0.6229	0.3054		0.4	0.6416	0.3050	0.6776	0.3020	0.6539	0.2808
-0.3	-0.4	0.2563	0.2127	0.2615	0.2009	0.2635	0.1952	0.2	-0.4	0.2811	0.2077	0.2855	0.1978	0.2881	0.1917
	-0.3	0.2552	0.2121	0.2554	0.2004	0.2569	0.1950		-0.3	0.2788	0.2074	0.2785	0.1976	0.2803	0.1919
	-0.2	0.2597	0.2108	0.2560	0.1991	0.2576	0.1940		-0.2	0.2817	0.2068	0.2774	0.1970	0.2787	0.1917
	-0.1	0.2660	0.2095	0.2597	0.1975	0.2617	0.1927		-0.1	0.2864	0.2060	0.2791	0.1960	0.2799	0.1913
	0.1	0.2698	0.2087	0.2645	0.1964	0.2665	0.1916		0.1	0.2884	0.2065	0.2772	0.1970	0.2722	0.1931
	0.2	0.2781	0.2095	0.2804	0.1961	0.2813	0.1914		0.2	0.2948	0.2094	0.2850	0.1981	0.2713	0.1952
	0.3	0.3496	0.2189	0.3699	0.2067	0.3665	0.2014		0.3	0.3624	0.2223	0.3651	0.2042	0.3407	0.1953
	0.4	0.6140	0.3119	0.6565	0.3155	0.6413	0.2995		0.4	0.6230	0.3162	0.6461	0.3024	0.6079	0.2738
-0.2	-0.4	0.2628	0.2090	0.2718	0.1981	0.2739	0.1920	0.3	-0.4	0.3564	0.2224	0.3430	0.2110	0.3461	0.2047
	-0.3	0.2559	0.2098	0.2604	0.1990	0.2619	0.1933		-0.3	0.3661	0.2194	0.3491	0.2077	0.3516	0.2014
	-0.2	0.2556	0.2099	0.2560	0.1991	0.2574	0.1940		-0.2	0.3776	0.2167	0.3580	0.2046	0.3600	0.1982
	-0.1	0.2585	0.2094	0.2562	0.1985	0.2578	0.1939		-0.1	0.3875	0.2147	0.3659	0.2020	0.3672	0.1956
	0.1	0.2629	0.2085	0.2614	0.1972	0.2628	0.1926		0.1	0.3872	0.2153	0.3600	0.2019	0.3539	0.1936
	0.2	0.2775	0.2078	0.2837	0.1951	0.2840	0.1905		0.2	0.3777	0.2221	0.3444	0.2072	0.3241	0.1971
	0.3	0.3589	0.2152	0.3816	0.2036	0.3778	0.1980		0.3	0.4028	0.2465	0.3705	0.2207	0.3185	0.2056
	0.4	0.6297	0.3067	0.6731	0.3107	0.6577	0.2943		0.4	0.6123	0.3490	0.6004	0.3122	0.5221	0.2647
-0.1	-0.4	0.2703	0.2063	0.2816	0.1960	0.2837	0.1896	0.4	-0.4	0.6380	0.3373	0.5964	0.3102	0.5997	0.3037
	-0.3	0.2598	0.2079	0.2669	0.1974	0.2684	0.1916		-0.3	0.6559	0.3319	0.6127	0.3044	0.6156	0.2979
	-0.2	0.2558	0.2089	0.2592	0.1985	0.2603	0.1934		-0.2	0.6724	0.3272	0.6281	0.2993	0.6306	0.2926
	-0.1	0.2561	0.2092	0.2565	0.1987	0.2577	0.1942		-0.1	0.6852	0.3237	0.6398	0.2952	0.6415	0.2880
	0.1	0.2608	0.2083	0.2617	0.1975	0.2623	0.1931		0.1	0.6834	0.3232	0.6323	0.2923	0.6262	0.2788
	0.2	0.2800	0.2065	0.2881	0.1942	0.2875	0.1896		0.2	0.6632	0.3308	0.6017	0.2958	0.5813	0.2718
	0.3	0.3676	0.2127	0.3910	0.2012	0.3864	0.1952		0.3	0.6456	0.3570	0.5632	0.3109	0.5010	0.2637
	0.4	0.6421	0.3032	0.6854	0.3067	0.6692	0.2897		0.4	0.7403	0.4545	0.6457	0.3765	0.4834	0.2866

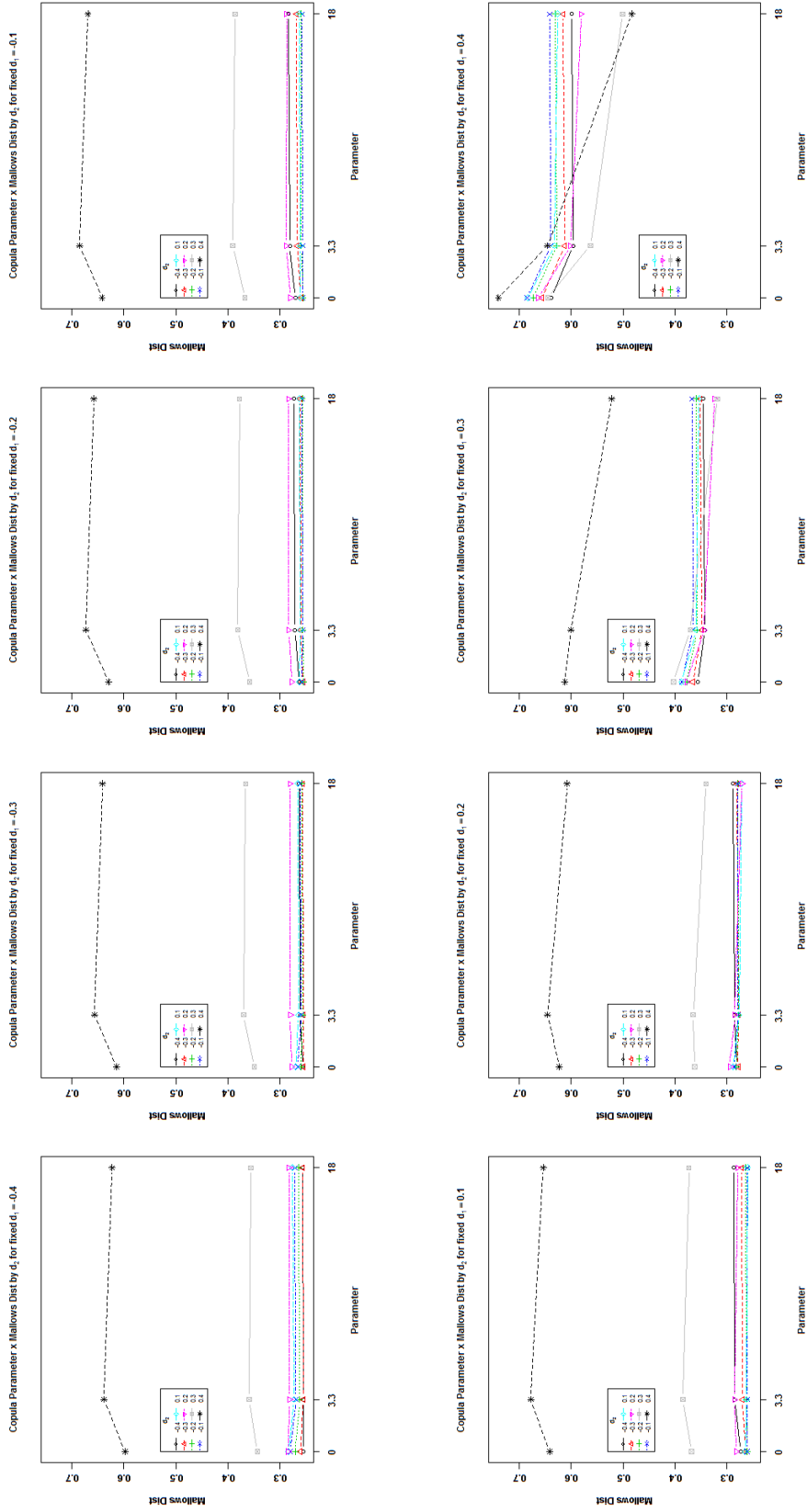


Figure 6: Plots of Mallows distance by the Frank copula’s parameter $\theta \in \{0, 3.3, 18\}$ for fixed d_1 . Marginals were taken to be (standardized)

Table 7: Simulation results for the Mallows distance in the Frank- t_7 case

$\epsilon_t \sim \text{Frank-}t_7$															
d		$\theta = 0$		$\theta = 3.3$		$\theta = 18$		d		$\theta = 0$		$\theta = 3.3$		$\theta = 18$	
d_1	d_2	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd	d_1	d_2	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd	$\hat{\mathcal{D}}_2$	sd
-0.4	-0.4	0.0841	0.0282	0.0839	0.0281	0.0796	0.0274	0.1	-0.4	0.1180	0.0299	0.1340	0.0288	0.1308	0.0269
	-0.3	0.0954	0.0289	0.0857	0.0283	0.0836	0.0268		-0.3	0.1006	0.0298	0.1111	0.0291	0.1078	0.0280
	-0.2	0.1122	0.0288	0.0973	0.0285	0.0974	0.0256		-0.2	0.0943	0.0297	0.0983	0.0295	0.0948	0.0288
	-0.1	0.1266	0.0283	0.1096	0.0283	0.1108	0.0247		-0.1	0.0951	0.0297	0.0932	0.0296	0.0889	0.0287
	0.1	0.1331	0.0291	0.1178	0.0296	0.1195	0.0274		0.1	0.1026	0.0346	0.0957	0.0305	0.0836	0.0284
	0.2	0.1375	0.0488	0.1331	0.0522	0.1326	0.0526		0.2	0.1314	0.0544	0.1323	0.0440	0.1153	0.0361
	0.3	0.2132	0.1208	0.2376	0.1267	0.2268	0.1265		0.3	0.2438	0.1135	0.2630	0.1048	0.2392	0.0938
	0.4	0.4974	0.2781	0.5527	0.2956	0.5221	0.2808		0.4	0.5430	0.2636	0.5875	0.2689	0.5460	0.2448
-0.3	-0.4	0.0864	0.0283	0.0959	0.0285	0.0914	0.0270	0.2	-0.4	0.1314	0.0498	0.1380	0.0483	0.1360	0.0477
	-0.3	0.0835	0.0283	0.0835	0.0282	0.0797	0.0276		-0.3	0.1271	0.0502	0.1261	0.0500	0.1244	0.0493
	-0.2	0.0918	0.0284	0.0840	0.0282	0.0821	0.0272		-0.2	0.1310	0.0492	0.1236	0.0500	0.1219	0.0486
	-0.1	0.1022	0.0280	0.0904	0.0281	0.0900	0.0262		-0.1	0.1372	0.0479	0.1253	0.0485	0.1225	0.0457
	0.1	0.1107	0.0293	0.1008	0.0297	0.1011	0.0287		0.1	0.1396	0.0516	0.1197	0.0451	0.1047	0.0345
	0.2	0.1262	0.0506	0.1295	0.0522	0.1270	0.0534		0.2	0.1494	0.0699	0.1298	0.0523	0.0959	0.0321
	0.3	0.2220	0.1178	0.2507	0.1222	0.2392	0.1217		0.3	0.2368	0.1272	0.2361	0.1042	0.1914	0.0757
	0.4	0.5158	0.2714	0.5721	0.2886	0.5413	0.2736		0.4	0.5252	0.2756	0.5525	0.2645	0.4924	0.2255
-0.2	-0.4	0.0982	0.0288	0.1130	0.0284	0.1089	0.0259	0.3	-0.4	0.2332	0.1249	0.2129	0.1208	0.2135	0.1199
	-0.3	0.0845	0.0283	0.0921	0.0282	0.0880	0.0272		-0.3	0.2460	0.1205	0.2212	0.1177	0.2220	0.1164
	-0.2	0.0833	0.0282	0.0832	0.0282	0.0799	0.0280		-0.2	0.2596	0.1160	0.2321	0.1135	0.2326	0.1115
	-0.1	0.0885	0.0280	0.0830	0.0282	0.0808	0.0276		-0.1	0.2699	0.1126	0.2406	0.1093	0.2397	0.1060
	0.1	0.0988	0.0299	0.0946	0.0296	0.0929	0.0294		0.1	0.2682	0.1132	0.2307	0.1027	0.2183	0.0889
	0.2	0.1246	0.0511	0.1334	0.0506	0.1293	0.0517		0.2	0.2572	0.1257	0.2080	0.1036	0.1718	0.0712
	0.3	0.2336	0.1142	0.2640	0.1175	0.2518	0.1163		0.3	0.2851	0.1725	0.2334	0.1297	0.1390	0.0612
	0.4	0.5319	0.2659	0.5883	0.2826	0.5571	0.2672		0.4	0.5131	0.3150	0.4941	0.2681	0.3786	0.1852
-0.1	-0.4	0.1105	0.0289	0.1275	0.0279	0.1237	0.0251	0.4	-0.4	0.5536	0.3010	0.4966	0.2784	0.4983	0.2775
	-0.3	0.0910	0.0284	0.1027	0.0279	0.0988	0.0264		-0.3	0.5729	0.2938	0.5148	0.2714	0.5163	0.2704
	-0.2	0.0835	0.0282	0.0885	0.0280	0.0847	0.0277		-0.2	0.5893	0.2879	0.5304	0.2654	0.5315	0.2639
	-0.1	0.0841	0.0281	0.0835	0.0281	0.0800	0.0282		-0.1	0.6006	0.2836	0.5410	0.2606	0.5408	0.2579
	0.1	0.0951	0.0304	0.0945	0.0293	0.0907	0.0290		0.1	0.5970	0.2828	0.5309	0.2537	0.5204	0.2416
	0.2	0.1278	0.0507	0.1387	0.0482	0.1328	0.0482		0.2	0.5766	0.2916	0.4968	0.2525	0.4674	0.2230
	0.3	0.2435	0.1114	0.2734	0.1130	0.2600	0.1107		0.3	0.5572	0.3225	0.4455	0.2619	0.3564	0.1853
	0.4	0.5436	0.2621	0.5989	0.2776	0.5666	0.2612		0.4	0.6505	0.4333	0.5163	0.3326	0.2659	0.1580

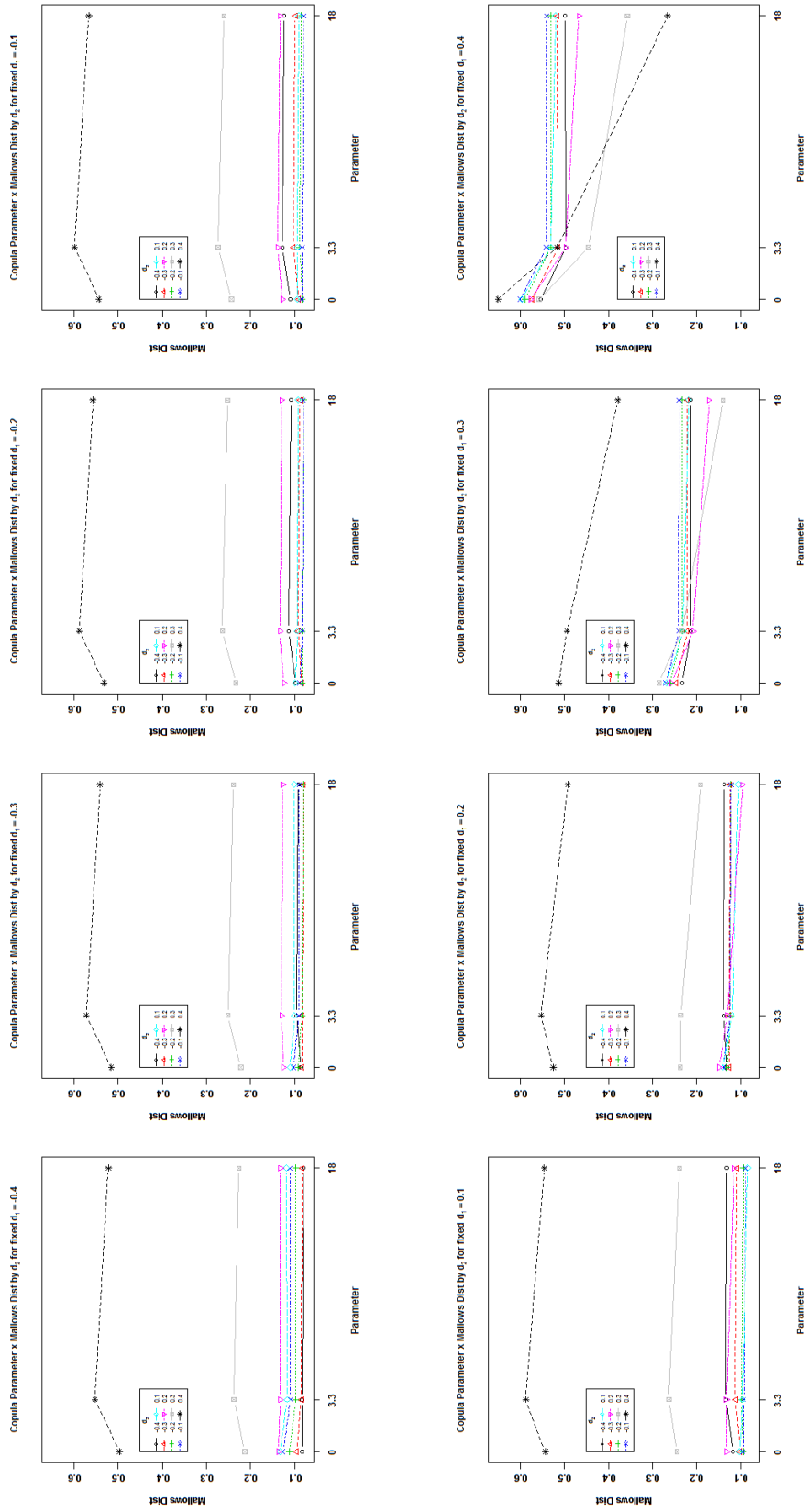


Figure 7: Plots of Mallows distance by the Frank copula’s parameter $\theta \in \{0, 3, 3, 18\}$ for fixed d_1 . Marginals were taken to be (standardized)

Table 8: Simulation results for the Kendall's τ in the Gaussian case, $\sigma^2 = (1, 1)$.

$\epsilon_t \sim \mathcal{N}_\rho(\mathbf{0}, \sigma^2), \sigma^2 = (1, 1)$															
\mathbf{d}		$\rho = 0$		$\rho = 0.5$		$\rho = 0.95$		\mathbf{d}		$\rho = 0$		$\rho = 0.5$		$\rho = 0.95$	
d_1	d_2	$\hat{\tau}$	sd	$\hat{\tau}$	sd	$\hat{\tau}$	sd	d_1	d_2	$\hat{\tau}$	sd	$\hat{\tau}$	sd	$\hat{\tau}$	sd
-0.4	-0.4	0.0000	0.0165	0.3347	0.0139	0.7987	0.0048	0.1	-0.4	0.0001	0.0147	0.2856	0.0125	0.6137	0.0071
	-0.3	0.0000	0.0162	0.3332	0.0138	0.7911	0.0048		-0.3	0.0000	0.0147	0.2991	0.0125	0.6559	0.0067
	-0.2	-0.0001	0.0159	0.3282	0.0136	0.7678	0.0050		-0.2	0.0000	0.0147	0.3117	0.0125	0.6999	0.0061
	-0.1	-0.0001	0.0156	0.3192	0.0134	0.7296	0.0054		-0.1	-0.0001	0.0148	0.3226	0.0125	0.7436	0.0054
	0.1	-0.0003	0.0147	0.2844	0.0128	0.6131	0.0072		0.1	-0.0001	0.0154	0.3335	0.0130	0.7977	0.0045
	0.2	-0.0004	0.0139	0.2561	0.0124	0.5359	0.0092		0.2	-0.0001	0.0160	0.3279	0.0136	0.7736	0.0053
	0.3	-0.0006	0.0127	0.2187	0.0124	0.4459	0.0129		0.3	-0.0001	0.0169	0.3092	0.0149	0.7010	0.0110
	0.4	-0.0007	0.0112	0.1726	0.0133	0.3455	0.0182		0.4	0.0001	0.0182	0.2730	0.0183	0.5903	0.0217
-0.3	-0.4	0.0000	0.0162	0.3333	0.0137	0.7913	0.0047	0.2	-0.4	0.0001	0.0139	0.2580	0.0122	0.5367	0.0093
	-0.3	0.0000	0.0160	0.3346	0.0136	0.7986	0.0047		-0.3	0.0000	0.0140	0.2735	0.0122	0.5789	0.0092
	-0.2	0.0000	0.0158	0.3328	0.0134	0.7897	0.0047		-0.2	0.0000	0.0142	0.2892	0.0123	0.6251	0.0089
	-0.1	-0.0001	0.0155	0.3269	0.0132	0.7622	0.0050		-0.1	-0.0001	0.0145	0.3045	0.0124	0.6750	0.0083
	0.1	-0.0003	0.0147	0.2979	0.0127	0.6553	0.0068		0.1	0.0000	0.0159	0.3289	0.0135	0.7745	0.0054
	0.2	-0.0004	0.0140	0.2716	0.0124	0.5780	0.0091		0.2	0.0002	0.0175	0.3329	0.0148	0.7972	0.0051
	0.3	-0.0006	0.0130	0.2352	0.0126	0.4855	0.0134		0.3	0.0006	0.0200	0.3248	0.0172	0.7630	0.0079
	0.4	-0.0007	0.0115	0.1884	0.0140	0.3804	0.0196		0.4	0.0013	0.0237	0.2986	0.0217	0.6680	0.0185
-0.2	-0.4	0.0000	0.0159	0.3285	0.0135	0.7681	0.0049	0.3	-0.4	0.0000	0.0128	0.2215	0.0121	0.4472	0.0131
	-0.3	0.0000	0.0158	0.3330	0.0133	0.7899	0.0046		-0.3	0.0000	0.0130	0.2382	0.0123	0.4870	0.0137
	-0.2	0.0000	0.0156	0.3345	0.0132	0.7985	0.0045		-0.2	-0.0001	0.0134	0.2559	0.0125	0.5318	0.0141
	-0.1	-0.0001	0.0154	0.3322	0.0131	0.7877	0.0046		-0.1	-0.0001	0.0140	0.2746	0.0129	0.5826	0.0142
	0.1	-0.0003	0.0148	0.3106	0.0127	0.6993	0.0061		0.1	0.0002	0.0168	0.3120	0.0147	0.7033	0.0112
	0.2	-0.0004	0.0142	0.2873	0.0125	0.6241	0.0088		0.2	0.0007	0.0199	0.3267	0.0171	0.7651	0.0078
	0.3	-0.0006	0.0134	0.2528	0.0128	0.5302	0.0138		0.3	0.0017	0.0250	0.3325	0.0215	0.7963	0.0077
	0.4	-0.0007	0.0122	0.2061	0.0147	0.4207	0.0211		0.4	0.0033	0.0325	0.3213	0.0283	0.7489	0.0140
-0.1	-0.4	0.0001	0.0156	0.3197	0.0132	0.7300	0.0052	0.4	-0.4	-0.0001	0.0113	0.1767	0.0129	0.3478	0.0183
	-0.3	0.0000	0.0155	0.3272	0.0131	0.7626	0.0048		-0.3	-0.0001	0.0117	0.1929	0.0134	0.3829	0.0198
	-0.2	0.0000	0.0154	0.3325	0.0130	0.7879	0.0045		-0.2	-0.0001	0.0123	0.2111	0.0140	0.4235	0.0212
	-0.1	-0.0001	0.0152	0.3343	0.0129	0.7983	0.0044		-0.1	-0.0001	0.0133	0.2314	0.0147	0.4711	0.0224
	0.1	-0.0003	0.0149	0.3217	0.0127	0.7430	0.0054		0.1	0.0004	0.0182	0.2786	0.0175	0.5946	0.0216
	0.2	-0.0004	0.0146	0.3027	0.0126	0.6740	0.0081		0.2	0.0013	0.0237	0.3036	0.0212	0.6729	0.0180
	0.3	-0.0005	0.0140	0.2713	0.0131	0.5808	0.0139		0.3	0.0032	0.0327	0.3248	0.0283	0.7536	0.0130
	0.4	-0.0007	0.0132	0.2261	0.0155	0.4679	0.0222		0.4	0.0062	0.0454	0.3332	0.0390	0.7950	0.0146

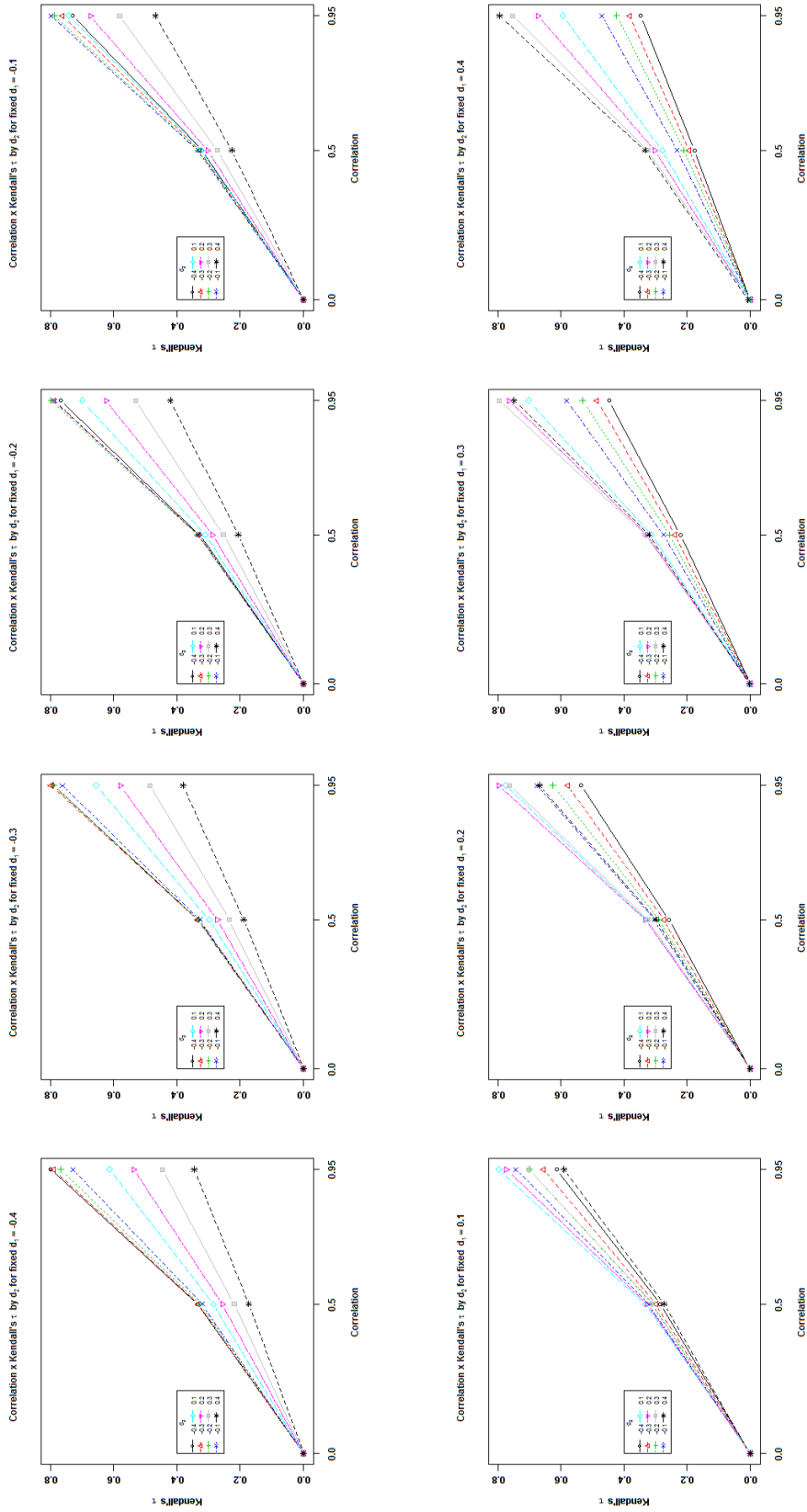


Figure 8: Plots of Kendall's τ by correlation for fixed d_1 . Gaussian case with equal variances.

Table 9: Simulation results for the Kendall's τ in the Gaussian case, $\sigma^2 = (1, 2)$.

$\epsilon_t \sim \mathcal{N}_\rho(\mathbf{0}, \sigma^2), \sigma^2 = (1, 2)$															
\mathbf{d}		$\rho = 0$		$\rho = 0.5$		$\rho = 0.95$		\mathbf{d}		$\rho = 0$		$\rho = 0.5$		$\rho = 0.95$	
d_1	d_2	$\hat{\tau}$	sd	$\hat{\tau}$	sd	$\hat{\tau}$	sd	d_1	d_2	$\hat{\tau}$	sd	$\hat{\tau}$	sd	$\hat{\tau}$	sd
-0.4	-0.4	0.0000	0.0152	0.0095	0.0269	0.0191	0.0569	0.1	-0.4	-0.0004	0.0144	0.0100	0.0262	0.0163	0.0422
	-0.3	0.0003	0.0157	0.0099	0.0290	0.0190	0.0515		-0.3	-0.0008	0.0138	0.0116	0.0330	0.0226	0.0547
	-0.2	0.0003	0.0157	0.0119	0.0328	0.0211	0.0544		-0.2	0.0000	0.0148	0.0087	0.0281	0.0170	0.0467
	-0.1	-0.0002	0.0153	0.0109	0.0330	0.0220	0.0599		-0.1	-0.0006	0.0150	0.0117	0.0298	0.0189	0.0444
	0.1	0.0001	0.0146	0.0093	0.0265	0.0155	0.0392		0.1	0.0010	0.0154	0.0143	0.0380	0.0315	0.0807
	0.2	-0.0007	0.0137	0.0085	0.0244	0.0192	0.0424		0.2	0.0022	0.0156	0.0143	0.0295	0.0263	0.0626
	0.3	-0.0002	0.0128	0.0057	0.0210	0.0137	0.0320		0.3	0.0029	0.0160	0.0186	0.0344	0.0228	0.0495
	0.4	0.0001	0.0103	0.0072	0.0192	0.0153	0.0356		0.4	0.0034	0.0191	0.0153	0.0294	0.0248	0.0481
-0.3	-0.4	-0.0009	0.0162	0.0111	0.0337	0.0329	0.0787	0.2	-0.4	0.0005	0.0136	0.0071	0.0205	0.0196	0.0547
	-0.3	0.0006	0.0158	0.0072	0.0283	0.0225	0.0583		-0.3	-0.0007	0.0137	0.0084	0.0236	0.0199	0.0496
	-0.2	-0.0006	0.0158	0.0094	0.0309	0.0193	0.0593		-0.2	-0.0001	0.0140	0.0135	0.0332	0.0206	0.0467
	-0.1	0.0004	0.0148	0.0114	0.0323	0.0226	0.0580		-0.1	0.0009	0.0143	0.0130	0.0345	0.0256	0.0644
	0.1	0.0001	0.0141	0.0104	0.0252	0.0188	0.0429		0.1	0.0022	0.0155	0.0154	0.0352	0.0297	0.0663
	0.2	0.0007	0.0137	0.0127	0.0298	0.0228	0.0631		0.2	0.0027	0.0173	0.0185	0.0391	0.0258	0.0595
	0.3	0.0009	0.0135	0.0079	0.0202	0.0135	0.0354		0.3	0.0046	0.0210	0.0233	0.0407	0.0290	0.0537
	0.4	0.0001	0.0116	0.0078	0.0193	0.0112	0.0292		0.4	0.0072	0.0246	0.0202	0.0323	0.0237	0.0517
-0.2	-0.4	0.0003	0.0156	0.0093	0.0262	0.0267	0.0671	0.3	-0.4	-0.0002	0.0119	0.0064	0.0196	0.0134	0.0328
	-0.3	0.0000	0.0156	0.0106	0.0263	0.0328	0.0803		-0.3	0.0001	0.0130	0.0092	0.0225	0.0140	0.0359
	-0.2	-0.0007	0.0151	0.0126	0.0334	0.0257	0.0622		-0.2	-0.0004	0.0135	0.0082	0.0226	0.0149	0.0400
	-0.1	-0.0002	0.0156	0.0125	0.0323	0.0220	0.0585		-0.1	0.0018	0.0145	0.0133	0.0298	0.0184	0.0479
	0.1	0.0001	0.0141	0.0097	0.0264	0.0221	0.0511		0.1	0.0028	0.0176	0.0146	0.0336	0.0279	0.0550
	0.2	0.0000	0.0141	0.0178	0.0369	0.0206	0.0476		0.2	0.0052	0.0198	0.0178	0.0350	0.0242	0.0429
	0.3	0.0003	0.0126	0.0090	0.0223	0.0151	0.0351		0.3	0.0090	0.0273	0.0232	0.0398	0.0369	0.0691
	0.4	0.0002	0.0120	0.0074	0.0184	0.0156	0.0384		0.4	0.0138	0.0343	0.0287	0.0427	0.0319	0.0640
-0.1	-0.4	0.0005	0.0150	0.0096	0.0270	0.0251	0.0695	0.4	-0.4	0.0000	0.0107	0.0083	0.0217	0.0109	0.0296
	-0.3	-0.0006	0.0156	0.0123	0.0303	0.0160	0.0450		-0.3	0.0007	0.0112	0.0072	0.0186	0.0090	0.0270
	-0.2	-0.0006	0.0155	0.0096	0.0289	0.0286	0.0784		-0.2	0.0006	0.0124	0.0116	0.0250	0.0128	0.0299
	-0.1	0.0006	0.0156	0.0097	0.0289	0.0221	0.0499		-0.1	0.0007	0.0128	0.0081	0.0208	0.0157	0.0412
	0.1	0.0006	0.0141	0.0120	0.0314	0.0217	0.0543		0.1	0.0047	0.0194	0.0134	0.0279	0.0227	0.0432
	0.2	0.0011	0.0150	0.0093	0.0240	0.0231	0.0508		0.2	0.0088	0.0258	0.0210	0.0427	0.0307	0.0538
	0.3	0.0001	0.0136	0.0125	0.0304	0.0229	0.0515		0.3	0.0102	0.0356	0.0276	0.0453	0.0410	0.0719
	0.4	0.0016	0.0134	0.0091	0.0229	0.0168	0.0374		0.4	0.0133	0.0557	0.0369	0.0623	0.0511	0.0821

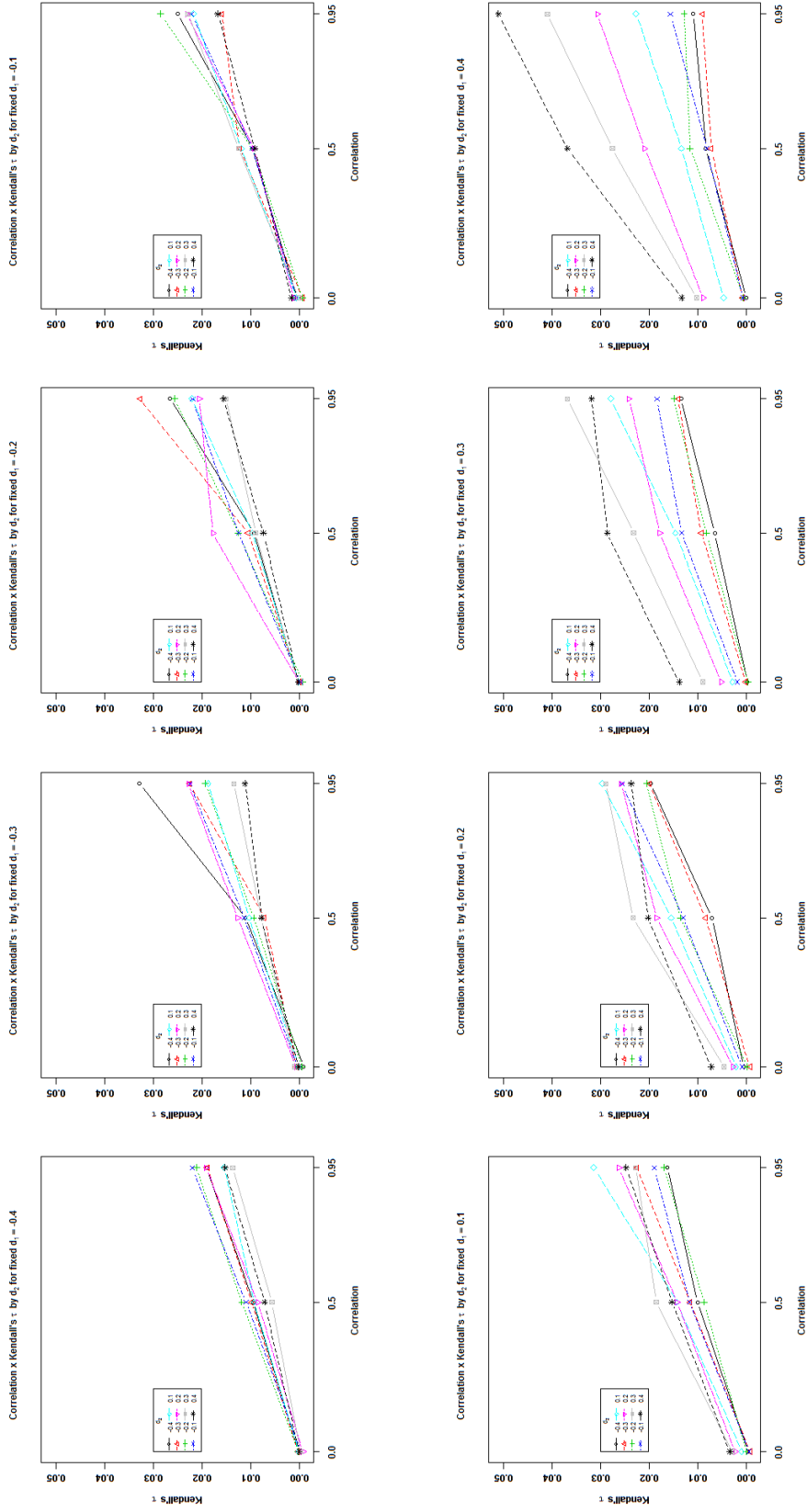


Figure 9: Plots of Kendall's τ by correlation for fixed d_1 and $\sigma^2 = (1, 2)$.

Table 10: Simulation results for the Kendall's τ in the Gaussian case, $\sigma^2 = (1, 3)$.

$\epsilon_t \sim \mathcal{N}_\rho(\mathbf{0}, \sigma^2), \sigma^2 = (1, 3)$															
\mathbf{d}		$\rho = 0$		$\rho = 0.5$		$\rho = 0.95$		\mathbf{d}		$\rho = 0$		$\rho = 0.5$		$\rho = 0.95$	
d_1	d_2	$\hat{\tau}$	sd	$\hat{\tau}$	sd	$\hat{\tau}$	sd	d_1	d_2	$\hat{\tau}$	sd	$\hat{\tau}$	sd	$\hat{\tau}$	sd
-0.4	-0.4	0.0011	0.0153	0.0085	0.0294	0.0203	0.0587	0.1	-0.4	-0.0006	0.0147	0.0119	0.0302	0.0151	0.0407
	-0.3	0.0004	0.0155	0.0083	0.0251	0.0197	0.0503		-0.3	0.0006	0.0137	0.0071	0.0241	0.0204	0.0511
	-0.2	-0.0003	0.0161	0.0128	0.0353	0.0230	0.0647		-0.2	0.0006	0.0150	0.0065	0.0238	0.0166	0.0505
	-0.1	-0.0008	0.0156	0.0128	0.0432	0.0234	0.0648		-0.1	0.0006	0.0148	0.0076	0.0269	0.0200	0.0533
	0.1	0.0000	0.0149	0.0093	0.0263	0.0135	0.0399		0.1	0.0013	0.0151	0.0117	0.0297	0.0222	0.0553
	0.2	-0.0003	0.0138	0.0096	0.0270	0.0179	0.0416		0.2	0.0025	0.0159	0.0108	0.0251	0.0213	0.0579
	0.3	0.0001	0.0126	0.0058	0.0199	0.0116	0.0326		0.3	0.0019	0.0165	0.0163	0.0326	0.0263	0.0564
	0.4	0.0004	0.0108	0.0076	0.0197	0.0146	0.0395		0.4	0.0040	0.0195	0.0136	0.0280	0.0213	0.0457
-0.3	-0.4	0.0006	0.0150	0.0143	0.0366	0.0247	0.0641	0.2	-0.4	0.0008	0.0133	0.0067	0.0225	0.0165	0.0589
	-0.3	-0.0003	0.0156	0.0102	0.0305	0.0188	0.0549		-0.3	0.0006	0.0137	0.0067	0.0221	0.0178	0.0480
	-0.2	-0.0002	0.0157	0.0085	0.0301	0.0201	0.0641		-0.2	-0.0004	0.0144	0.0098	0.0259	0.0156	0.0436
	-0.1	-0.0006	0.0152	0.0095	0.0294	0.0153	0.0517		-0.1	0.0010	0.0145	0.0089	0.0265	0.0256	0.0652
	0.1	-0.0003	0.0145	0.0089	0.0275	0.0227	0.0632		0.1	0.0023	0.0160	0.0154	0.0362	0.0238	0.0618
	0.2	0.0006	0.0136	0.0127	0.0291	0.0174	0.0469		0.2	0.0020	0.0174	0.0154	0.0347	0.0203	0.0587
	0.3	-0.0008	0.0128	0.0052	0.0199	0.0127	0.0389		0.3	0.0055	0.0203	0.0163	0.0333	0.0254	0.0526
	0.4	0.0001	0.0117	0.0071	0.0210	0.0100	0.0300		0.4	0.0055	0.0252	0.0166	0.0326	0.0236	0.0511
-0.2	-0.4	0.0007	0.0156	0.0095	0.0255	0.0147	0.0433	0.3	-0.4	-0.0011	0.0124	0.0054	0.0202	0.0120	0.0335
	-0.3	0.0001	0.0155	0.0086	0.0278	0.0292	0.0869		-0.3	0.0004	0.0129	0.0085	0.0235	0.0124	0.0366
	-0.2	-0.0002	0.0154	0.0085	0.0276	0.0277	0.0750		-0.2	0.0002	0.0130	0.0078	0.0220	0.0175	0.0464
	-0.1	0.0010	0.0161	0.0101	0.0329	0.0206	0.0574		-0.1	0.0017	0.0144	0.0100	0.0261	0.0172	0.0458
	0.1	0.0008	0.0142	0.0094	0.0278	0.0193	0.0478		0.1	0.0024	0.0171	0.0140	0.0303	0.0230	0.0487
	0.2	-0.0004	0.0142	0.0180	0.0357	0.0200	0.0497		0.2	0.0053	0.0214	0.0209	0.0412	0.0228	0.0492
	0.3	0.0002	0.0125	0.0083	0.0224	0.0135	0.0375		0.3	0.0085	0.0261	0.0229	0.0372	0.0301	0.0573
	0.4	0.0011	0.0116	0.0075	0.0202	0.0123	0.0389		0.4	0.0126	0.0376	0.0257	0.0440	0.0368	0.0685
-0.1	-0.4	0.0005	0.0150	0.0093	0.0275	0.0179	0.0529	0.4	-0.4	0.0000	0.0103	0.0072	0.0196	0.0088	0.0284
	-0.3	0.0002	0.0154	0.0116	0.0313	0.0174	0.0516		-0.3	-0.0001	0.0110	0.0062	0.0176	0.0103	0.0304
	-0.2	-0.0001	0.0156	0.0092	0.0282	0.0282	0.0823		-0.2	0.0004	0.0115	0.0092	0.0228	0.0145	0.0419
	-0.1	-0.0004	0.0150	0.0074	0.0270	0.0201	0.0532		-0.1	0.0009	0.0129	0.0106	0.0249	0.0137	0.0331
	0.1	-0.0005	0.0147	0.0110	0.0308	0.0239	0.0611		0.1	0.0035	0.0179	0.0115	0.0263	0.0210	0.0437
	0.2	0.0006	0.0134	0.0094	0.0239	0.0200	0.0473		0.2	0.0088	0.0243	0.0177	0.0361	0.0259	0.0483
	0.3	0.0003	0.0140	0.0101	0.0295	0.0199	0.0481		0.3	0.0097	0.0373	0.0257	0.0426	0.0411	0.0661
	0.4	0.0011	0.0131	0.0070	0.0212	0.0163	0.0381		0.4	0.0128	0.0513	0.0327	0.0610	0.0452	0.0733

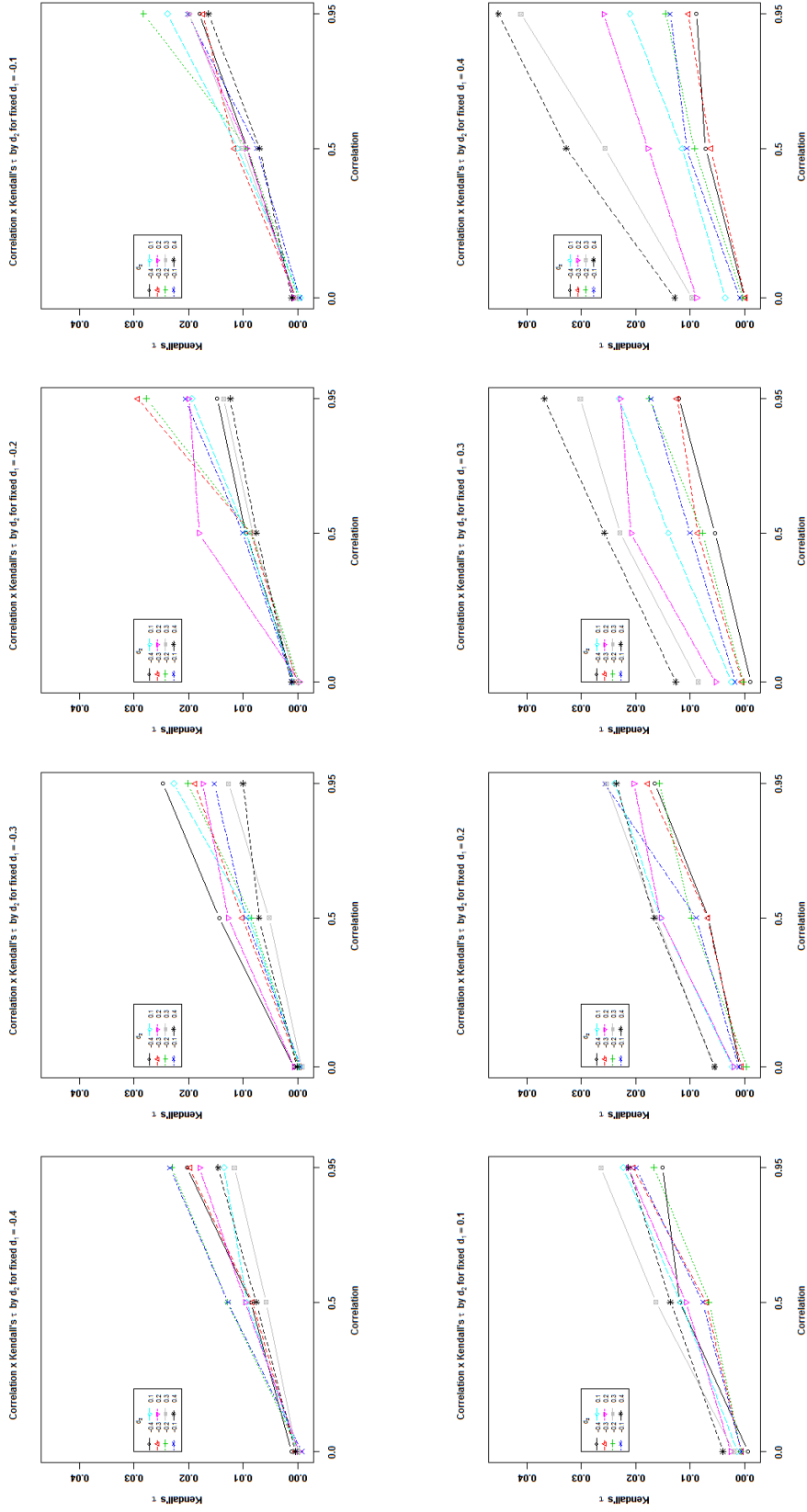


Figure 10: Plots of Kendall's τ by correlation for fixed d_1 and $\sigma^2 = (1, 3)$.

Table 11: Simulation results for the Kendall's τ in the Gaussian case, $\sigma^2 = (2, 3)$.

$\epsilon_t \sim \mathcal{N}_\rho(\mathbf{0}, \sigma^2), \sigma^2 = (2, 3)$															
\mathbf{d}		$\rho = 0$		$\rho = 0.5$		$\rho = 0.95$		\mathbf{d}		$\rho = 0$		$\rho = 0.5$		$\rho = 0.95$	
d_1	d_2	$\hat{\tau}$	sd	$\hat{\tau}$	sd	$\hat{\tau}$	sd	d_1	d_2	$\hat{\tau}$	sd	$\hat{\tau}$	sd	$\hat{\tau}$	sd
-0.4	-0.4	0.0007	0.0158	0.0089	0.0300	0.0199	0.0607	0.1	-0.4	0.0007	0.0141	0.0088	0.0274	0.0155	0.0445
	-0.3	0.0001	0.0157	0.0079	0.0290	0.0194	0.0552		-0.3	0.0004	0.0140	0.0074	0.0260	0.0185	0.0529
	-0.2	0.0004	0.0154	0.0113	0.0351	0.0210	0.0621		-0.2	0.0002	0.0145	0.0069	0.0261	0.0161	0.0483
	-0.1	0.0005	0.0153	0.0079	0.0272	0.0232	0.0691		-0.1	0.0009	0.0143	0.0095	0.0282	0.0173	0.0509
	0.1	0.0002	0.0147	0.0079	0.0267	0.0152	0.0459		0.1	0.0005	0.0151	0.0117	0.0309	0.0201	0.0535
	0.2	0.0002	0.0135	0.0087	0.0265	0.0171	0.0406		0.2	0.0027	0.0156	0.0108	0.0278	0.0297	0.0690
	0.3	0.0004	0.0117	0.0055	0.0200	0.0116	0.0354		0.3	0.0021	0.0166	0.0185	0.0355	0.0224	0.0533
	0.4	0.0002	0.0107	0.0049	0.0166	0.0143	0.0358		0.4	0.0037	0.0190	0.0134	0.0270	0.0228	0.0507
-0.3	-0.4	0.0001	0.0159	0.0096	0.0307	0.0221	0.0623	0.2	-0.4	0.0000	0.0135	0.0077	0.0238	0.0118	0.0358
	-0.3	0.0005	0.0160	0.0087	0.0310	0.0186	0.0574		-0.3	0.0001	0.0139	0.0072	0.0235	0.0178	0.0478
	-0.2	-0.0005	0.0162	0.0110	0.0330	0.0204	0.0687		-0.2	-0.0001	0.0140	0.0079	0.0261	0.0153	0.0447
	-0.1	0.0004	0.0158	0.0090	0.0301	0.0180	0.0525		-0.1	0.0009	0.0140	0.0100	0.0293	0.0209	0.0543
	0.1	0.0010	0.0151	0.0101	0.0285	0.0227	0.0546		0.1	0.0016	0.0161	0.0109	0.0278	0.0232	0.0644
	0.2	0.0007	0.0137	0.0115	0.0295	0.0164	0.0434		0.2	0.0032	0.0178	0.0139	0.0347	0.0227	0.0613
	0.3	0.0005	0.0125	0.0070	0.0224	0.0106	0.0372		0.3	0.0049	0.0206	0.0172	0.0310	0.0231	0.0514
	0.4	0.0004	0.0114	0.0063	0.0196	0.0107	0.0335		0.4	0.0070	0.0251	0.0183	0.0347	0.0242	0.0541
-0.2	-0.4	0.0000	0.0154	0.0095	0.0274	0.0192	0.0549	0.3	-0.4	0.0000	0.0127	0.0065	0.0200	0.0109	0.0318
	-0.3	-0.0003	0.0156	0.0094	0.0281	0.0190	0.0561		-0.3	0.0007	0.0123	0.0089	0.0240	0.0118	0.0359
	-0.2	0.0008	0.0158	0.0103	0.0309	0.0257	0.0636		-0.2	0.0003	0.0131	0.0079	0.0232	0.0163	0.0444
	-0.1	-0.0010	0.0151	0.0099	0.0298	0.0242	0.0620		-0.1	0.0023	0.0139	0.0107	0.0262	0.0147	0.0434
	0.1	0.0000	0.0145	0.0103	0.0277	0.0190	0.0539		0.1	0.0027	0.0168	0.0145	0.0317	0.0241	0.0522
	0.2	0.0000	0.0134	0.0150	0.0342	0.0209	0.0556		0.2	0.0043	0.0207	0.0191	0.0357	0.0237	0.0495
	0.3	-0.0005	0.0132	0.0084	0.0223	0.0151	0.0404		0.3	0.0081	0.0259	0.0222	0.0360	0.0306	0.0602
	0.4	0.0002	0.0116	0.0066	0.0202	0.0146	0.0408		0.4	0.0105	0.0370	0.0280	0.0437	0.0294	0.0646
-0.1	-0.4	-0.0002	0.0153	0.0117	0.0301	0.0168	0.0492	0.4	-0.4	-0.0002	0.0104	0.0064	0.0183	0.0095	0.0290
	-0.3	-0.0004	0.0153	0.0104	0.0304	0.0163	0.0546		-0.3	0.0005	0.0112	0.0047	0.0167	0.0100	0.0330
	-0.2	-0.0001	0.0151	0.0086	0.0283	0.0145	0.0464		-0.2	0.0002	0.0115	0.0085	0.0234	0.0117	0.0302
	-0.1	-0.0003	0.0145	0.0080	0.0303	0.0215	0.0553		-0.1	0.0012	0.0129	0.0090	0.0214	0.0129	0.0323
	0.1	0.0001	0.0148	0.0123	0.0356	0.0173	0.0518		0.1	0.0052	0.0184	0.0119	0.0268	0.0205	0.0435
	0.2	0.0011	0.0145	0.0098	0.0262	0.0215	0.0537		0.2	0.0076	0.0251	0.0189	0.0355	0.0273	0.0575
	0.3	0.0005	0.0140	0.0116	0.0297	0.0195	0.0462		0.3	0.0112	0.0367	0.0246	0.0439	0.0344	0.0641
	0.4	0.0009	0.0128	0.0089	0.0217	0.0149	0.0378		0.4	0.0133	0.0553	0.0306	0.0607	0.0515	0.0840

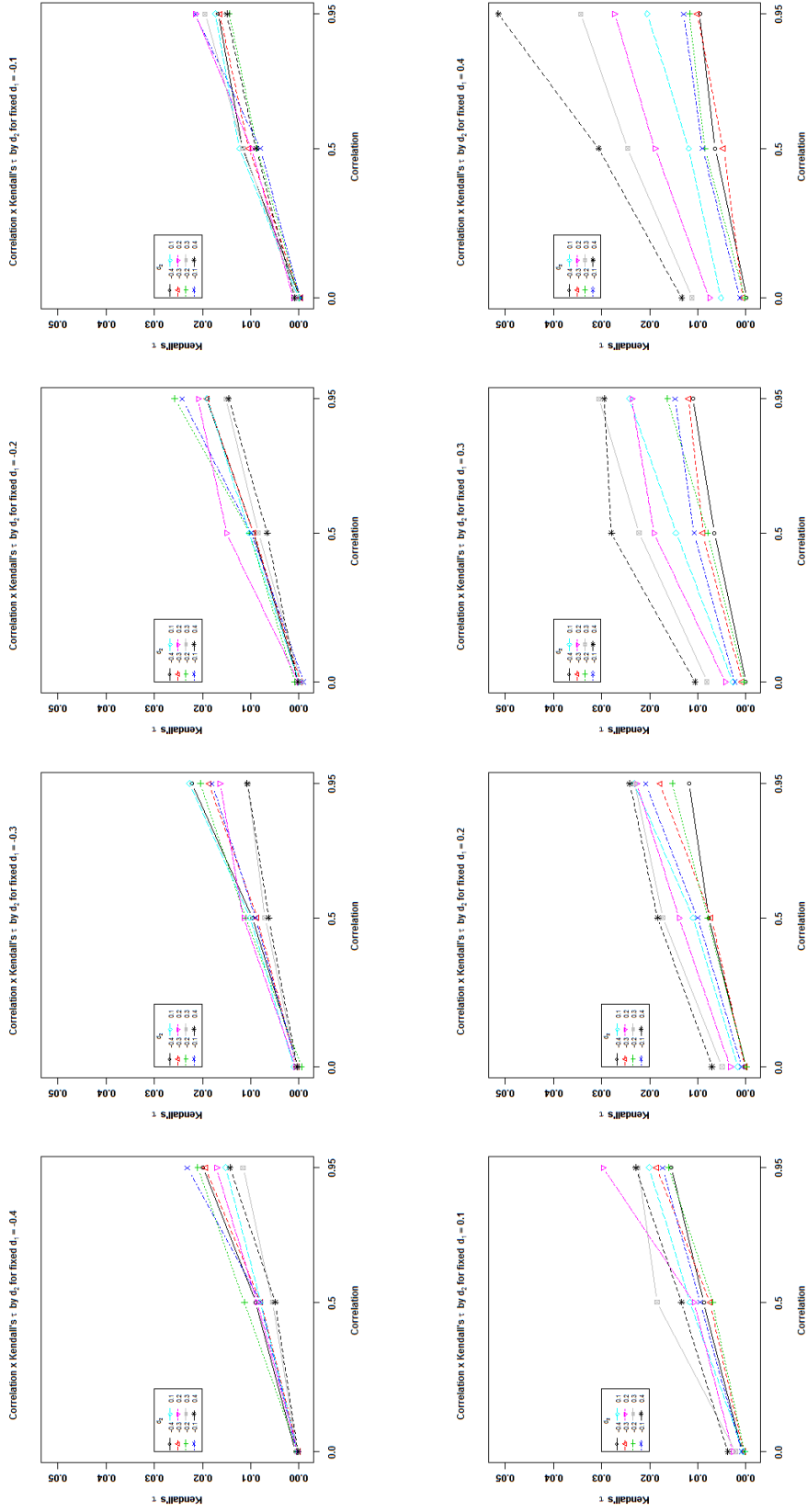


Figure 11: Plots of Kendall's τ by correlation for fixed d_1 and $\sigma^2 = (2, 3)$.

Table 12: Simulation results for the Kendall's τ in the Frank- $\mathcal{N}(0, 1)$ case

$\varepsilon_t \sim \text{Frank-}\mathcal{N}(0, 1)$															
\mathbf{d}		$\theta = 0$		$\theta = 3.3$		$\theta = 18$		\mathbf{d}		$\theta = 0$		$\theta = 3.3$		$\theta = 18$	
d_1	d_2	$\hat{\tau}$	sd	$\hat{\tau}$	sd	$\hat{\tau}$	sd	d_1	d_2	$\hat{\tau}$	sd	$\hat{\tau}$	sd	$\hat{\tau}$	sd
-0.4	-0.4	0.0059	0.0159	0.3238	0.0136	0.7770	0.0047	0.1	-0.4	0.0048	0.0140	0.2823	0.0123	0.6066	0.0071
	-0.3	0.0060	0.0157	0.3235	0.0134	0.7731	0.0046		-0.3	0.0050	0.0141	0.2963	0.0122	0.6493	0.0065
	-0.2	0.0061	0.0155	0.3200	0.0131	0.7542	0.0047		-0.2	0.0054	0.0141	0.3095	0.0121	0.6940	0.0057
	-0.1	0.0062	0.0152	0.3125	0.0128	0.7193	0.0051		-0.1	0.0058	0.0142	0.3208	0.0120	0.7387	0.0048
	0.1	0.0063	0.0143	0.2801	0.0121	0.6057	0.0072		0.1	0.0074	0.0146	0.3312	0.0123	0.7928	0.0038
	0.2	0.0063	0.0135	0.2522	0.0119	0.5288	0.0098		0.2	0.0085	0.0151	0.3243	0.0129	0.7637	0.0057
	0.3	0.0059	0.0125	0.2149	0.0123	0.4394	0.0141		0.3	0.0097	0.0162	0.3036	0.0144	0.6883	0.0125
	0.4	0.0052	0.0111	0.1692	0.0135	0.3408	0.0194		0.4	0.0106	0.0180	0.2662	0.0175	0.5796	0.0228
-0.3	-0.4	0.0059	0.0157	0.3238	0.0134	0.7730	0.0046	0.2	-0.4	0.0042	0.0133	0.2553	0.0121	0.5308	0.0094
	-0.3	0.0060	0.0155	0.3263	0.0132	0.7826	0.0043		-0.3	0.0044	0.0134	0.2713	0.0120	0.5733	0.0092
	-0.2	0.0061	0.0153	0.3257	0.0130	0.7772	0.0043		-0.2	0.0048	0.0136	0.2873	0.0119	0.6198	0.0088
	-0.1	0.0063	0.0151	0.3211	0.0127	0.7529	0.0045		-0.1	0.0053	0.0139	0.3028	0.0119	0.6698	0.0079
	0.1	0.0066	0.0143	0.2941	0.0121	0.6481	0.0067		0.1	0.0073	0.0152	0.3262	0.0128	0.7669	0.0049
	0.2	0.0067	0.0136	0.2681	0.0119	0.5711	0.0097		0.2	0.0089	0.0168	0.3284	0.0143	0.7836	0.0052
	0.3	0.0065	0.0126	0.2314	0.0125	0.4791	0.0147		0.3	0.0109	0.0196	0.3177	0.0169	0.7439	0.0094
	0.4	0.0059	0.0114	0.1849	0.0140	0.3757	0.0209		0.4	0.0126	0.0241	0.2895	0.0211	0.6508	0.0195
-0.2	-0.4	0.0057	0.0154	0.3206	0.0132	0.7541	0.0047	0.3	-0.4	0.0034	0.0122	0.2191	0.0122	0.4426	0.0133
	-0.3	0.0059	0.0153	0.3261	0.0130	0.7772	0.0042		-0.3	0.0036	0.0125	0.2360	0.0122	0.4827	0.0137
	-0.2	0.0061	0.0151	0.3288	0.0128	0.7884	0.0040		-0.2	0.0040	0.0128	0.2539	0.0123	0.5277	0.0140
	-0.1	0.0063	0.0149	0.3276	0.0125	0.7802	0.0040		-0.1	0.0045	0.0134	0.2725	0.0123	0.5782	0.0138
	0.1	0.0069	0.0143	0.3074	0.0120	0.6926	0.0060		0.1	0.0070	0.0162	0.3084	0.0139	0.6949	0.0105
	0.2	0.0071	0.0137	0.2841	0.0120	0.6171	0.0094		0.2	0.0091	0.0196	0.3210	0.0168	0.7495	0.0081
	0.3	0.0071	0.0129	0.2489	0.0127	0.5234	0.0152		0.3	0.0119	0.0253	0.3234	0.0219	0.7686	0.0094
	0.4	0.0067	0.0119	0.2024	0.0146	0.4158	0.0224		0.4	0.0147	0.0337	0.3090	0.0288	0.7193	0.0156
-0.1	-0.4	0.0055	0.0150	0.3135	0.0129	0.7193	0.0051	0.4	-0.4	0.0025	0.0108	0.1745	0.0130	0.3449	0.0184
	-0.3	0.0057	0.0149	0.3220	0.0127	0.7529	0.0044		-0.3	0.0026	0.0112	0.1909	0.0134	0.3804	0.0197
	-0.2	0.0060	0.0148	0.3282	0.0126	0.7804	0.0039		-0.2	0.0029	0.0118	0.2091	0.0137	0.4212	0.0209
	-0.1	0.0063	0.0147	0.3309	0.0124	0.7933	0.0037		-0.1	0.0034	0.0128	0.2292	0.0141	0.4686	0.0217
	0.1	0.0071	0.0143	0.3191	0.0120	0.7371	0.0052		0.1	0.0061	0.0178	0.2745	0.0165	0.5886	0.0202
	0.2	0.0076	0.0139	0.2996	0.0120	0.6666	0.0087		0.2	0.0087	0.0236	0.2972	0.0212	0.6604	0.0171
	0.3	0.0078	0.0134	0.2673	0.0129	0.5732	0.0153		0.3	0.0121	0.0331	0.3142	0.0297	0.7270	0.0154
	0.4	0.0077	0.0128	0.2218	0.0152	0.4622	0.0235		0.4	0.0160	0.0465	0.3176	0.0410	0.7510	0.0187

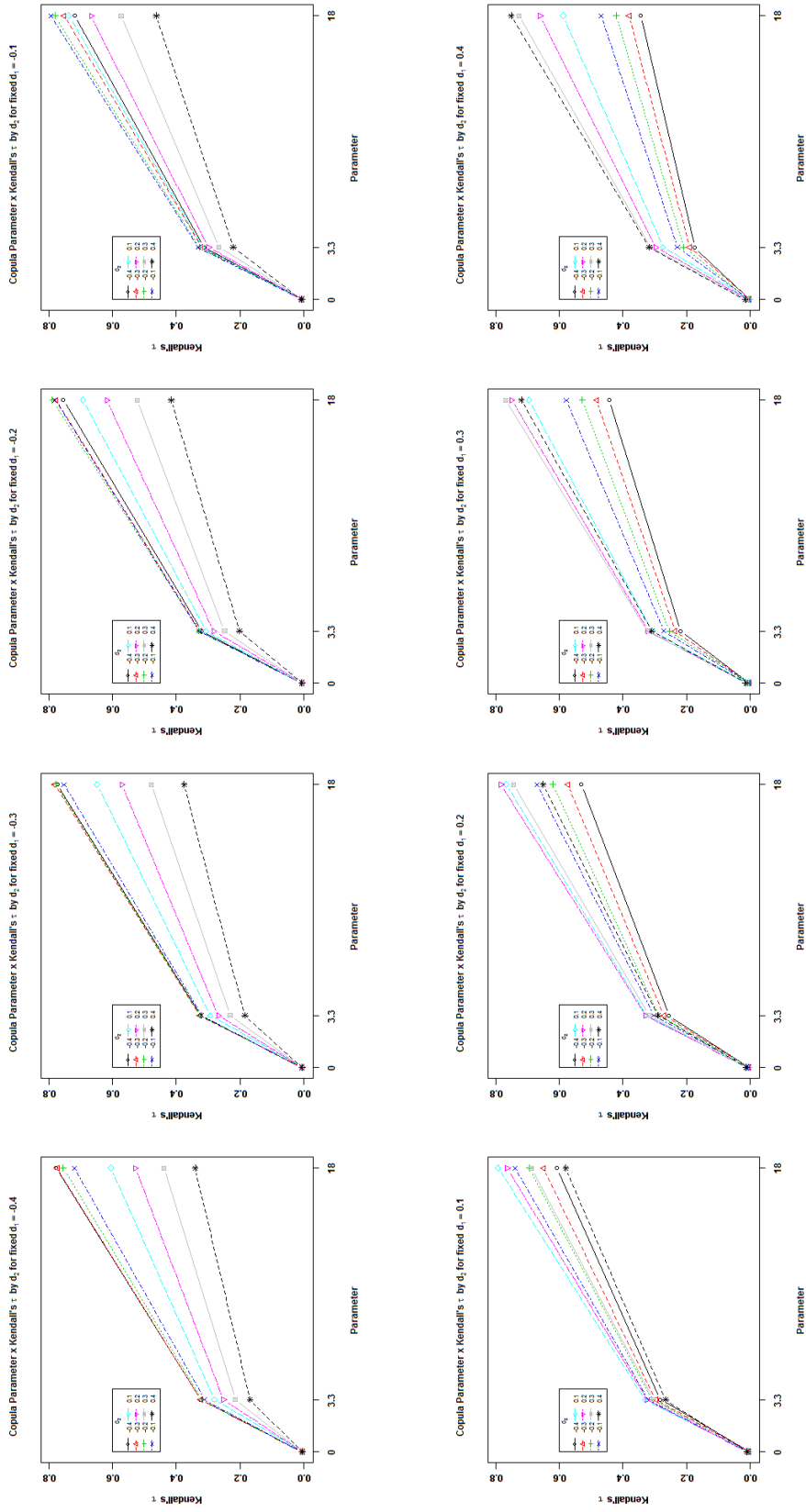


Figure 12: Plots of Kendall's τ by the Frank copula's parameter $\theta \in \{0, 3, 3, 18\}$ for fixed d_1 . Marginals were taken to be standard normal.

Table 13: Simulation results for the Kendall's τ in the Frank- t_3 case

$\varepsilon_t \sim \text{Frank-}t_3$															
d		$\theta = 0$		$\theta = 3.3$		$\theta = 18$		d		$\theta = 0$		$\theta = 3.3$		$\theta = 18$	
d_1	d_2	$\hat{\tau}$	sd	$\hat{\tau}$	sd	$\hat{\tau}$	sd	d_1	d_2	$\hat{\tau}$	sd	$\hat{\tau}$	sd	$\hat{\tau}$	sd
-0.4	-0.4	0.0022	0.0572	0.3048	0.0729	0.7346	0.0954	0.1	-0.4	0.0013	0.0567	0.2598	0.0703	0.5530	0.0858
	-0.3	0.0023	0.0572	0.3058	0.0729	0.7340	0.0954		-0.3	0.0016	0.0567	0.2756	0.0711	0.5956	0.0880
	-0.2	0.0025	0.0572	0.3041	0.0728	0.7212	0.0947		-0.2	0.0020	0.0568	0.2924	0.0720	0.6454	0.0907
	-0.1	0.0027	0.0571	0.2977	0.0724	0.6897	0.0930		-0.1	0.0026	0.0568	0.3089	0.0728	0.7021	0.0936
	0.1	0.0029	0.0569	0.2571	0.0701	0.5523	0.0857		0.1	0.0042	0.0571	0.3227	0.0737	0.7724	0.0974
	0.2	0.0025	0.0566	0.2212	0.0682	0.4593	0.0811		0.2	0.0050	0.0573	0.3094	0.0732	0.7230	0.0952
	0.3	0.0019	0.0564	0.1783	0.0660	0.3608	0.0766		0.3	0.0055	0.0579	0.2805	0.0722	0.6287	0.0914
	0.4	0.0010	0.0561	0.1328	0.0637	0.2640	0.0723		0.4	0.0057	0.0586	0.2382	0.0710	0.5135	0.0875
-0.3	-0.4	0.0022	0.0572	0.3061	0.0730	0.7336	0.0954	0.2	-0.4	0.0006	0.0565	0.2242	0.0684	0.4617	0.0812
	-0.3	0.0024	0.0572	0.3098	0.0731	0.7441	0.0959		-0.3	0.0008	0.0565	0.2413	0.0693	0.5028	0.0834
	-0.2	0.0026	0.0571	0.3110	0.0731	0.7438	0.0959		-0.2	0.0013	0.0566	0.2606	0.0703	0.5523	0.0861
	-0.1	0.0029	0.0571	0.3079	0.0729	0.7237	0.0948		-0.1	0.0019	0.0567	0.2812	0.0714	0.6115	0.0892
	0.1	0.0032	0.0569	0.2730	0.0709	0.5949	0.0880		0.1	0.0038	0.0573	0.3109	0.0732	0.7281	0.0954
	0.2	0.0030	0.0567	0.2383	0.0691	0.5002	0.0834		0.2	0.0049	0.0582	0.3087	0.0737	0.7315	0.0958
	0.3	0.0024	0.0564	0.1951	0.0669	0.3977	0.0788		0.3	0.0061	0.0597	0.2910	0.0739	0.6745	0.0939
	0.4	0.0016	0.0562	0.1479	0.0647	0.2953	0.0744		0.4	0.0068	0.0621	0.2578	0.0742	0.5766	0.0911
-0.2	-0.4	0.0022	0.0571	0.3049	0.0729	0.7202	0.0946	0.3	-0.4	-0.0002	0.0562	0.1815	0.0662	0.3645	0.0766
	-0.3	0.0024	0.0571	0.3116	0.0732	0.7431	0.0959		-0.3	0.0000	0.0562	0.1986	0.0671	0.4019	0.0788
	-0.2	0.0027	0.0571	0.3163	0.0734	0.7586	0.0967		-0.2	0.0003	0.0563	0.2187	0.0682	0.4481	0.0815
	-0.1	0.0030	0.0571	0.3172	0.0734	0.7560	0.0965		-0.1	0.0009	0.0565	0.2415	0.0694	0.5051	0.0846
	0.1	0.0036	0.0569	0.2901	0.0718	0.6443	0.0906		0.1	0.0031	0.0577	0.2841	0.0723	0.6380	0.0917
	0.2	0.0035	0.0567	0.2576	0.0701	0.5492	0.0860		0.2	0.0046	0.0596	0.2934	0.0741	0.6815	0.0943
	0.3	0.0031	0.0565	0.2150	0.0681	0.4431	0.0814		0.3	0.0063	0.0631	0.2891	0.0763	0.6779	0.0953
	0.4	0.0024	0.0563	0.1664	0.0658	0.3349	0.0771		0.4	0.0077	0.0686	0.2690	0.0794	0.6197	0.0948
-0.1	-0.4	0.0021	0.0570	0.2992	0.0725	0.6884	0.0929	0.4	-0.4	-0.0011	0.0559	0.1360	0.0638	0.2681	0.0723
	-0.3	0.0023	0.0570	0.3091	0.0730	0.7225	0.0947		-0.3	-0.0010	0.0559	0.1516	0.0648	0.3000	0.0745
	-0.2	0.0027	0.0570	0.3179	0.0734	0.7552	0.0965		-0.2	-0.0007	0.0561	0.1706	0.0659	0.3405	0.0771
	-0.1	0.0031	0.0570	0.3236	0.0737	0.7771	0.0976		-0.1	-0.0002	0.0563	0.1934	0.0673	0.3921	0.0803
	0.1	0.0040	0.0569	0.3071	0.0727	0.7003	0.0936		0.1	0.0020	0.0583	0.2438	0.0713	0.5246	0.0880
	0.2	0.0041	0.0568	0.2784	0.0713	0.6074	0.0891		0.2	0.0038	0.0618	0.2629	0.0747	0.5875	0.0920
	0.3	0.0039	0.0567	0.2376	0.0693	0.4988	0.0845		0.3	0.0059	0.0683	0.2722	0.0799	0.6266	0.0957
	0.4	0.0034	0.0565	0.1887	0.0672	0.3850	0.0802		0.4	0.0079	0.0783	0.2676	0.0872	0.6214	0.0988

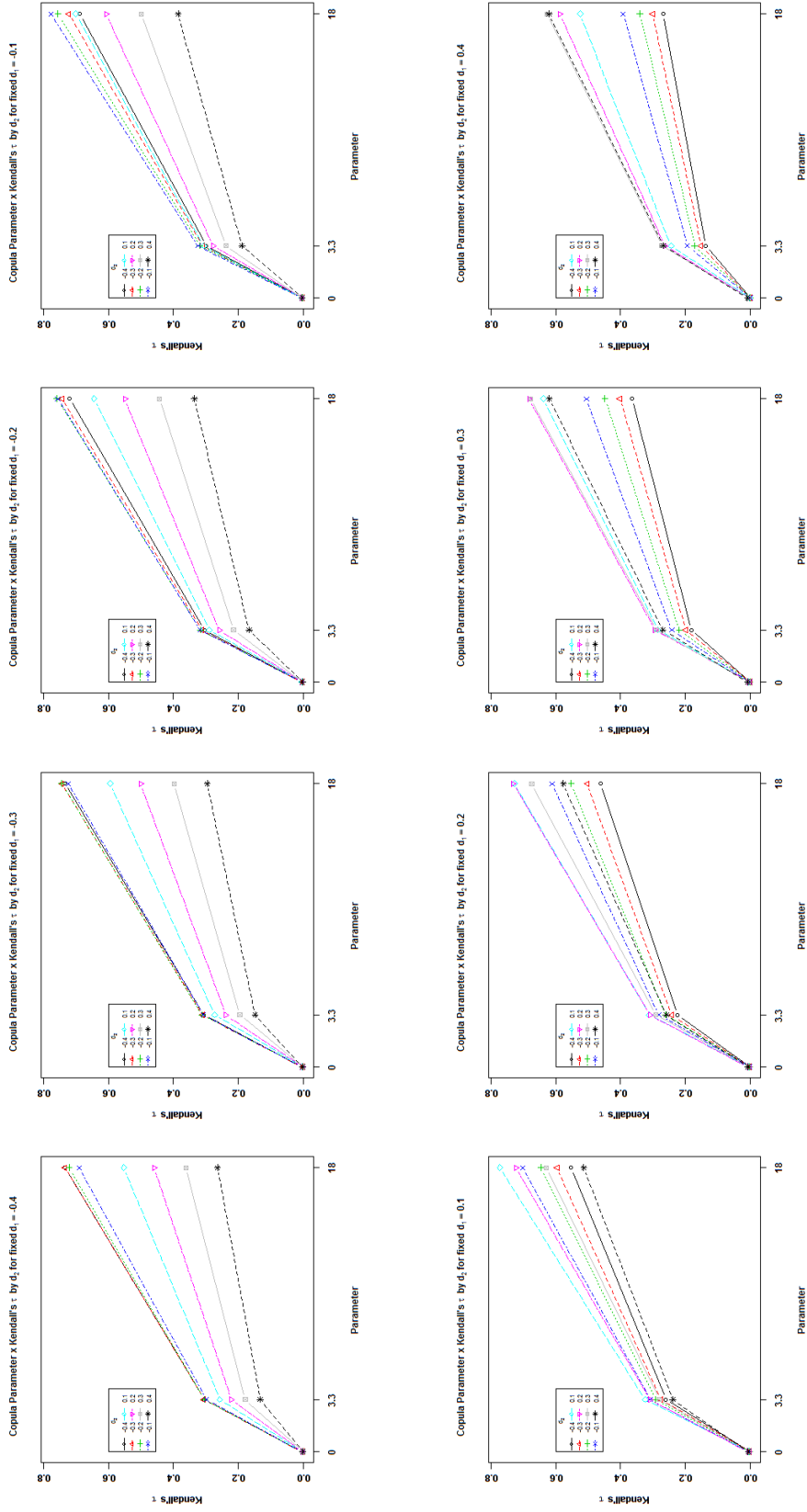


Figure 13: Plots of Kendall's τ by the Frank copula's parameter $\theta \in \{0, 3.3, 18\}$ for fixed d_1 . Marginals were taken to be (standardized)

Table 14: Simulation results for the Kendall's τ in the Frank- t_7 case

$\varepsilon_t \sim \text{Frank-}t_7$															
d		$\theta = 0$		$\theta = 3.3$		$\theta = 18$		d		$\theta = 0$		$\theta = 3.3$		$\theta = 18$	
d_1	d_2	$\hat{\tau}$	sd	$\hat{\tau}$	sd	$\hat{\tau}$	sd	d_1	d_2	$\hat{\tau}$	sd	$\hat{\tau}$	sd	$\hat{\tau}$	sd
-0.4	-0.4	0.0026	0.0573	0.3151	0.0735	0.7596	0.0967	0.1	-0.4	0.0017	0.0567	0.2730	0.0710	0.5872	0.0874
	-0.3	0.0028	0.0572	0.3154	0.0734	0.7573	0.0966		-0.3	0.0019	0.0568	0.2879	0.0717	0.6303	0.0898
	-0.2	0.0029	0.0572	0.3125	0.0732	0.7406	0.0957		-0.2	0.0023	0.0568	0.3023	0.0725	0.6771	0.0922
	-0.1	0.0031	0.0571	0.3052	0.0728	0.7063	0.0938		-0.1	0.0028	0.0568	0.3151	0.0732	0.7255	0.0948
	0.1	0.0032	0.0569	0.2706	0.0708	0.5864	0.0874		0.1	0.0043	0.0570	0.3263	0.0738	0.7846	0.0980
	0.2	0.0030	0.0567	0.2406	0.0691	0.5053	0.0833		0.2	0.0053	0.0572	0.3179	0.0735	0.7501	0.0963
	0.3	0.0026	0.0564	0.2016	0.0671	0.4134	0.0789		0.3	0.0063	0.0576	0.2950	0.0726	0.6694	0.0926
	0.4	0.0018	0.0561	0.1561	0.0648	0.3157	0.0747		0.4	0.0070	0.0582	0.2565	0.0713	0.5593	0.0886
-0.3	-0.4	0.0026	0.0572	0.3158	0.0735	0.7571	0.0966	0.2	-0.4	0.0010	0.0565	0.2437	0.0693	0.5073	0.0833
	-0.3	0.0028	0.0572	0.3188	0.0736	0.7676	0.0971		-0.3	0.0012	0.0566	0.2604	0.0702	0.5499	0.0856
	-0.2	0.0030	0.0571	0.3189	0.0735	0.7641	0.0969		-0.2	0.0016	0.0566	0.2776	0.0711	0.5978	0.0882
	-0.1	0.0032	0.0571	0.3146	0.0733	0.7407	0.0957		-0.1	0.0022	0.0567	0.2945	0.0720	0.6507	0.0910
	0.1	0.0035	0.0569	0.2855	0.0716	0.6294	0.0897		0.1	0.0041	0.0572	0.3196	0.0736	0.7538	0.0964
	0.2	0.0035	0.0567	0.2571	0.0700	0.5476	0.0856		0.2	0.0056	0.0578	0.3208	0.0740	0.7677	0.0972
	0.3	0.0032	0.0565	0.2186	0.0681	0.4528	0.0812		0.3	0.0073	0.0589	0.3084	0.0739	0.7231	0.0952
	0.4	0.0025	0.0562	0.1720	0.0658	0.3500	0.0769		0.4	0.0087	0.0607	0.2790	0.0736	0.6291	0.0916
-0.2	-0.4	0.0025	0.0571	0.3133	0.0733	0.7402	0.0956	0.3	-0.4	0.0002	0.0562	0.2056	0.0673	0.4168	0.0790
	-0.3	0.0027	0.0571	0.3194	0.0736	0.7638	0.0969		-0.3	0.0004	0.0563	0.2229	0.0682	0.4565	0.0812
	-0.2	0.0030	0.0571	0.3228	0.0737	0.7769	0.0976		-0.2	0.0008	0.0564	0.2417	0.0693	0.5024	0.0837
	-0.1	0.0032	0.0571	0.3221	0.0736	0.7702	0.0972		-0.1	0.0014	0.0566	0.2616	0.0704	0.5551	0.0866
	0.1	0.0038	0.0569	0.3001	0.0724	0.6758	0.0922		0.1	0.0037	0.0575	0.2994	0.0727	0.6768	0.0928
	0.2	0.0039	0.0568	0.2743	0.0710	0.5950	0.0881		0.2	0.0057	0.0588	0.3113	0.0741	0.7292	0.0954
	0.3	0.0038	0.0566	0.2371	0.0691	0.4979	0.0837		0.3	0.0080	0.0613	0.3121	0.0756	0.7420	0.0963
	0.4	0.0033	0.0563	0.1901	0.0669	0.3905	0.0795		0.4	0.0104	0.0656	0.2963	0.0774	0.6907	0.0946
-0.1	-0.4	0.0024	0.0570	0.3064	0.0729	0.7059	0.0938	0.4	-0.4	-0.0007	0.0559	0.1608	0.0650	0.3197	0.0747
	-0.3	0.0026	0.0570	0.3156	0.0733	0.7403	0.0956		-0.3	-0.0006	0.0560	0.1772	0.0660	0.3547	0.0769
	-0.2	0.0029	0.0570	0.3227	0.0737	0.7701	0.0972		-0.2	-0.0003	0.0561	0.1959	0.0671	0.3960	0.0794
	-0.1	0.0032	0.0570	0.3263	0.0738	0.7859	0.0981		-0.1	0.0002	0.0563	0.2170	0.0683	0.4450	0.0823
	0.1	0.0041	0.0569	0.3133	0.0731	0.7239	0.0948		0.1	0.0027	0.0579	0.2638	0.0715	0.5690	0.0886
	0.2	0.0044	0.0568	0.2914	0.0719	0.6473	0.0909		0.2	0.0051	0.0603	0.2858	0.0740	0.6393	0.0919
	0.3	0.0046	0.0567	0.2568	0.0702	0.5497	0.0865		0.3	0.0081	0.0652	0.3008	0.0780	0.6983	0.0951
	0.4	0.0043	0.0566	0.2105	0.0682	0.4384	0.0823		0.4	0.0113	0.0735	0.3022	0.0835	0.7131	0.0968

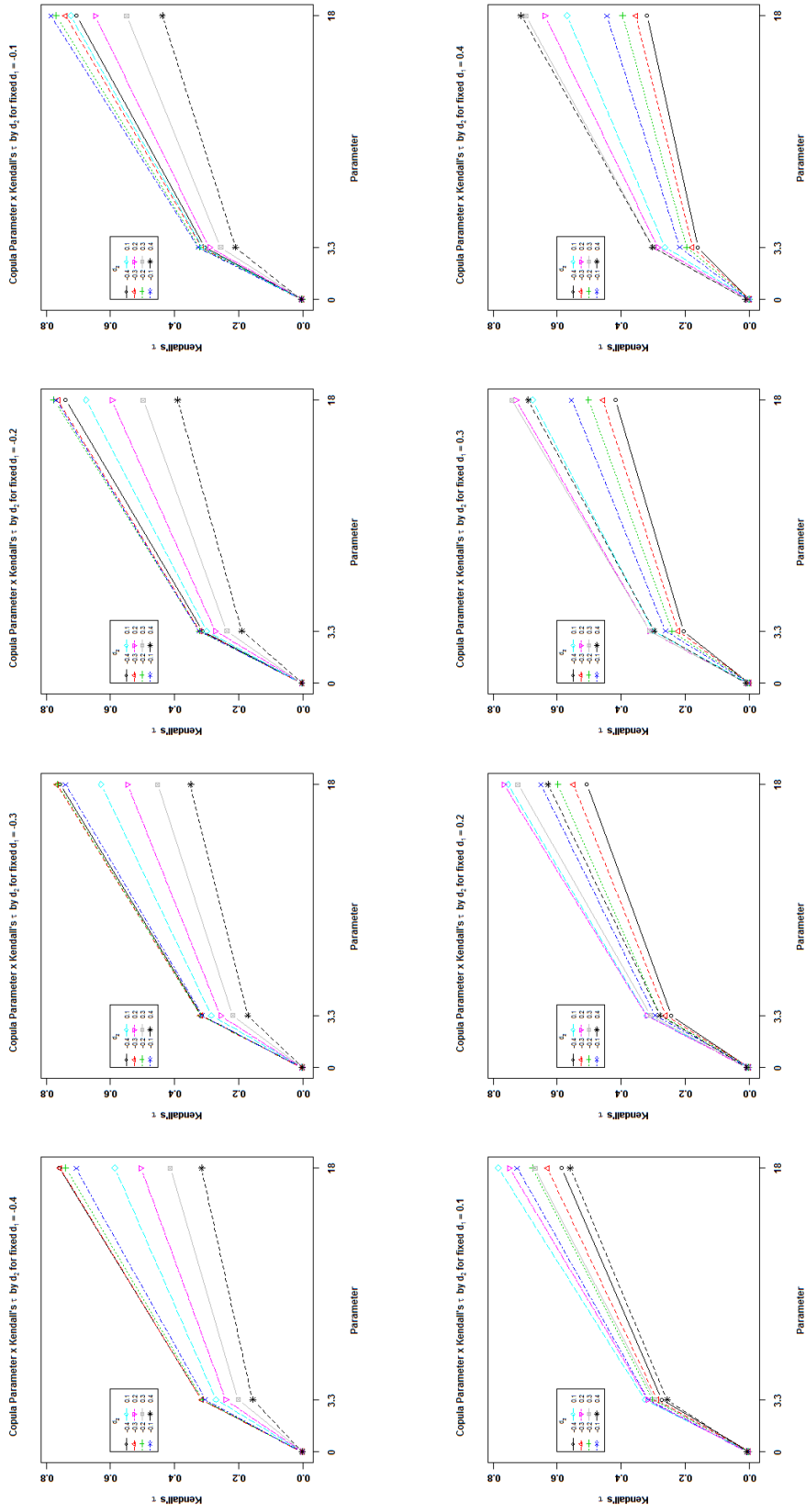


Figure 14: Plots of Kendall's τ by the Frank copula's parameter $\theta \in \{0, 3.3, 18\}$ for fixed d_1 . Marginals were taken to be (standardized)

t_7 .